1. Error Analysis

(a) Explain the difference between systematic and random errors.

(b) Calculate the value and the uncertainty of the slope associated to the data presented in fig. 1.

(c) Assuming that you have

\[ I(x) = I_0 e^{-\mu \sqrt{x}}, \]

derive the expression that you would use in order to calculate the relative uncertainty \( \frac{dT}{T} \) as a function of the uncertainties \( \frac{dT_0}{I_0}, \frac{d\mu}{\mu}, \) and \( \frac{dx}{x}. \)

Figure 1: Error Analysis
2. Lock-in Amplifier
   (a) Before using a lock-in amplifier, it is generally necessary to adjust or check some of the instrument’s settings in order to optimize its performance. What are the basic settings and how would you go about adjusting or choosing them?
   (b) Briefly comment on the kind of experimental situation in which a lock-in amplifier might be of use and explain why it would be useful.

3. Laser
   (a) Briefly explain the meaning of the following terms as they are applied to the operation of a laser:
      i. spontaneous and stimulated emission,
      ii. population inversion
      iii. coherent emission
   (b) Explain the role of the laser cavity.

4. X-Ray Diffraction
   (a) Consider X-rays of wavelength $\lambda$ incident on a crystal containing planes of atoms separated by a distance $d$. Derive the Bragg condition for angle $\theta$ at which the scattered X-rays interfere constructively. Assume that $\theta$ is measured from the scattering plane (see fig. 2).
   (b) Assume that the X-rays described in part (a) have a wavelength of 0.090 nm and are incident on a crystal with atoms residing on a cubic lattice in which the cube edge lengths are 0.135 nm. Calculate the lattice spacing and X-ray reflection angles for the families of planes having Miller indices of (100) and (101).

![Diagram of X-Ray Diffraction](image-url)
ANSWER ANY TWO OF THE FOLLOWING QUESTIONS

5. Diffraction

(a) Explain the difference between Fresnel and Fraunhofer diffraction.
(b) Given the single slit diffraction pattern

\[ I(\beta) = I_o \left( \frac{\sin^2 \beta}{\beta^2} \right) \]

show that the maxima in \( I(\beta) \) occur when \( \tan \beta = \beta \).
(c) Explain the physical significance of each factor in the double slit diffraction profile given by

\[ I(\theta) = I_o \left( \frac{\sin^2 \beta}{\beta^2} \right) \cos^2 \alpha \]

where

\[ \beta = \frac{2\pi b}{\lambda} \sin \theta \quad \text{and} \quad \alpha = \frac{2\pi a}{\lambda} \sin \theta \]

Here, \( a \) and \( b \) are the distance between the slits and the slit width, respectively.

6. Polarization

(a) Consider a beam of unpolarized light, of intensity \( I_o \), passing through three polarizers whose transmission axes are oriented at some angle with respect to each other. In a case where the first and the second polarizer are respectively at 0° and 20°, derive an expression for the intensity of the light transmitted by the third polarizer when it is at an angle \( \theta \) relative to the first one.

(b) i. Explain what is at the origin of Rayleigh scattering.
   ii. Under which conditions can we observe Rayleigh scattering?
   iii. What are the characteristics of Rayleigh scattering?

7. Gamma Ray

(a) Sketch a diagram of a typical gamma ray spectrum and label the important features.
(b) Describe the basic ways in which the energy carried by \( \gamma \)-rays can be partially or fully absorbed in a solid.
(c) The spectrum of a radioactive source shows three peaks on a multi-channel analyzer. It is known that the peak with channel number 938 corresponds to a gamma ray of energy 1.2 MeV and the one with channel number 360 corresponds to another gamma ray of energy 0.52 MeV. What is the energy of the gamma ray corresponding to the third peak with a channel number of 649?
(d) What is the uncertainty in a counting experiment?
8. Debye Temperature

(a) Using the equipartition of energy theorem, calculate the classical limit of the internal energy of a solid. Using your previous result, show that classically the heat capacity of a solid is given by $C_v = 3R$, where $R$ is the gas constant.

(b) Briefly describe the Debye model for the specific heat.

9. Michelson Interferometer

(a) Figure 3 shows a schematic diagram of the Michelson interferometer.
   
   i. Reproduce this diagram in your exam booklet and identify the different elements.
   
   ii. For the semi-transparent mirror, indicate the side that has a reflecting coating.
   
   iii. Draw the path followed by light.
   
   iv. What is the role of the CP element in the Michelson interferometer?

(b) If a gas cell is introduced in one path of the Michelson interferometer (see fig. 3), derive the relation between the number of fringes $\Delta m$ passing a given point as a function of the index of refraction $n_g$ of the gas, the wavelength of the monochromatic light source and the cell's length $d$.

![Schematic Diagram of the Michelson Interferometer](image)

Figure 3: Schematic Diagram of the Michelson Interferometer
10. **Drag**

(a) Consider a sphere of mass $m$ attached to a rotating vertical spindle of radius $a$ by a light string. The sphere travels in a circle of radius $r$ with a constant speed $v$. Show that the drag force on the sphere is given by (the string is tangential to the spindle, see figure 4)

$$D(v) = \frac{mv^2a}{r\sqrt{r^2 - a^2}}$$

(b) Define Reynolds number and drag coefficient

![Figure 4: Schematic diagram of the Drag experiment](image)

11. **Speed of Light**

Consider a measurement of the speed of light using a laser, a rotating mirror and a fixed mirror as shown below. Derive a relationship between the lateral displacement (for small displacement) of the reflected beam $d$ and the rotational period of the mirror $T$ in terms of the speed of light $c$, the round trip distance $L$, and the distance $r$ from the laser to the rotating mirror.

![Figure 5: Schematic diagram of the speed of light experiment](image)
12. Vortex Dipole

(a) A geometrical characteristic of a dipole, \( \alpha \), is defined as \( \alpha = D/L \), where \( D \) is the width of the dipole and \( L \) is its length. \( \alpha \) is controlled by the intensity of the source and viscosity of the fluid and can be presented as a function of three dimensional arguments

\[
\alpha = f(q, d, \nu)
\]

where \( q \) is the volume flux from the nozzle, \( d \) is the diameter of the nozzle and \( \nu \) is the kinematic viscosity of the fluid (dimensions \( cm^2/s \)). Use the dimensional analysis to reduce the number of arguments of this function from three to one non-dimensional argument.

(b) Describe briefly the technique used to measure the velocity field in this experiment.