Final Examination, April 2002  
Course: PHYS 3900  
Date of Examination: April 24, 2002  
Time of Examination: 9:00 - 11:00

No Examination aids other than those specified on this examination script are permitted.

ANSWER ALL QUESTIONS

1. Error Analysis

   (a) Explain the difference between systematic and random errors.

   (b) A student measures the length of a table to be \( L = 3.00 \pm 0.05 \text{ m} \) and its width to be \( 1.00 \pm 0.05 \text{ m} \). What is the uncertainty in its area? Show your work.

   (c) Assuming that you have

   \[
   v = \frac{2\pi L}{\ln(a/b)}
   \]

   where

   \[
   \begin{align*}
   L &= 5.2 \pm 0.2 \text{ m} \\
   a &= 2.4 \pm 0.2 \text{ cm} \\
   b &= 1.5 \pm 0.2 \text{ cm},
   \end{align*}
   \]

   calculate the relative uncertainty in \( v \).

2. Lock-in Amplifier

   (a) Before using a lock-in amplifier, it is generally necessary to adjust or check some of the instrument's settings in order to optimize its performance. What are the basic settings and how would you go about adjusting or choosing them?

   (b) Briefly comment on the kind of experimental situation in which a lock-in amplifier might be of use and explain why it would be useful.
3. **Laser**

   (a) Briefly explain the meaning of the following terms as they are applied to the operation of a laser:
   
   i. spontaneous and stimulated emission,
   
   ii. population inversion
   
   iii. coherent emission

   (b) Explain the role of the laser cavity.

4. **Inelastic Laser Light Scattering**

   (a) What is a phonon?

   (b) Briefly describe the phenomena of Brillouin and Raman scattering.
ANSWER ANY THREE OF THE FOLLOWING QUESTIONS

5. Diffraction

(a) Explain the difference between Fresnel and Fraunhofer diffraction.
(b) Given the single slit diffraction pattern

\[ I(\beta) = I_o \left( \frac{\sin^2 \beta}{\beta^2} \right), \]

show that the maxima in \( I(\beta) \) occur when \( \tan \beta = \beta \).
(c) Explain the physical significance of each factor in the double slit diffraction profile given by

\[ I(\theta) = I_o \left( \frac{\sin \beta}{\beta^2} \right) \cos^2 \alpha \]

where

\[ \beta = \frac{2\pi b}{\lambda} \sin \theta \quad \text{and} \quad \alpha = \frac{2\pi a}{\lambda} \sin \theta \]

Here, \( a \) and \( b \) are the distance between the slits and the slit width, respectively.

6. Polarization

(a) Consider a beam of unpolarized light, of intensity \( I_o \), passing through three polarizers whose transmission axes are oriented at some angle with respect to each other. In a case where the first and the second polarizer are respectively at 0° and 30°, derive an expression for the intensity of the light transmitted by the third polarizer when it is at an angle \( \theta \) relative to the first one.

7. Gamma Ray

(a) Sketch a diagram of a typical gamma ray spectrum and label the important features.
(b) Describe the basic ways in which the energy carried by \( \gamma \)-rays can be partially or fully absorbed in a solid.
(c) The spectrum of a radioactive source shows three peaks on a multi-channel analyzer. It is known that the peak with channel number 738 corresponds to a gamma ray of energy 1.3 MeV and the one with channel number 560 corresponds to another gamma ray of energy 0.62 MeV. What is the energy of the gamma ray corresponding to the third peak with a channel number of 649?
(d) What is the uncertainty in a counting experiment?
8. Debye Temperature

(a) Using the equipartition of energy theorem, calculate the classical limit of the internal energy of a solid. Using your previous result, show that classically the heat capacity of a solid is given by $C_v = 3R$, where $R$ is the gas constant.

(b) Briefly describe the Debye model for the specific heat.

9. Grating

(a) Derive the grating equation for plane waves of wavelength $\lambda$ incident normally upon a grating for which the distance between slits is $a$ (see figure 1).

(b) In the diffraction grating experiment, you measured the light spectrum of a few light sources either by looking at the first or second order diffraction pattern using the equation

$$n\lambda = a \sin \theta_n$$

where $\lambda$ is the wavelength of the diffracted light, $\theta_n$ is the diffraction angle relative to the grating normal, $a$ is the grating constant, and $n$ is the diffraction order. Sometimes, lines that are easily observed in the first order diffraction pattern are missing in the second order, even if they are very intense. Explain why or under what conditions this can happen.

![Figure 1: Schematic diagram of a Grating](image)
10. **Michelson Interferometer**

(a) Figure 2 shows a schematic diagram of the Michelson interferometer.

i. Reproduce this diagram in your exam booklet and identify the different elements.

ii. For the semi-transparent mirror, indicate the side that has a reflecting coating.

iii. Draw the path followed by light.

iv. What is the role of the CP element in the Michelson interferometer?

(b) For the interference pattern obtained when using monochromatic light of wavelength $\lambda$, derive an expression for the number of fringes which pass a given point when one of the mirrors is moved a distance $d$.

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Figure 2: Schematic Diagram of the Michelson Interferometer
11. **Drag**

(a) Consider a sphere of mass $m$ attached to a rotating vertical spindle of radius $a$ by a light string. The sphere travels in a circle of radius $r$ with a constant speed $v$. Show that the drag force on the sphere is given by (the string is tangential to the spindle, see figure 3)

$$D(v) = \frac{mv^2a}{r\sqrt{r^2 - a^2}}$$

(b) Define Reynolds number and drag coefficient

![Figure 3: Schematic diagram of the Drag experiment](image)

12. **Speed of Light**

Consider a measurement of the speed of light using a laser, a rotating mirror and a fixed mirror as shown below. Derive a relationship between the lateral displacement (for small displacement) of the reflected beam $d$ and the rotational period of the mirror $T$ in terms of the speed of light $c$, the round trip distance $L$, and the distance $r$ from the laser to the rotating mirror.

![Figure 4: Schematic diagram of the speed of light experiment](image)