MEMORIAL UNIVERSITY OF NEWFOUNDLAND
DEPARTMENT OF PHYSICS AND PHYSICAL OCEANOGRAPHY

PHYSICS 3600 FINAL EXAMINATION – WINTER 2006 – APRIL 19, 2006

NAME:_________________________STUDENT NUMBER:____________________________

INSTRUCTIONS:

1. Put your name and student number on each page.
2. Do any 5 of the 6 questions.
3. Each question is worth 20 marks.
4. Equations and constants are provided on the next page.
5. Use only the paper provided. No other books, notes or papers are permitted.
6. Do not remove examination papers from the examination room.
CONSTANTS AND FORMULAE

\[ c = 2.998 \times 10^8 \text{ m/s} \]
\[ n_1 \sin \theta_1 = n_2 \sin \theta_2 \]

\[ \frac{1}{s} + \frac{1}{s'} = \frac{1}{f} \]
\[ f = -\frac{R}{2} \]
\[ m = \frac{s'}{s} \]

\[ \frac{n_1}{s} + \frac{n_2}{s'} = \frac{n_2 - n_1}{R} \]
\[ m = \frac{n_1 s'}{n_2 s} \]

\[ \frac{1}{s} + \frac{1}{s'} = \frac{1}{f} \]
\[ \frac{1}{f} = \frac{n_2 - n_1}{n_1} \left( \frac{1}{R_1} - \frac{1}{R_2} \right) \]
\[ m = \frac{h_1}{h_2} = -\frac{s'}{s} \]

\[ I = I_1 + I_2 + 2 \sqrt{I_1 I_2 \cos \delta} \]

\[ I = 4I_0 \cos^2 \left( \frac{n \Delta}{\lambda} \right) = 4I_0 \cos^2 \left( \frac{n \alpha}{\lambda s} \right) \]

\[ I = I_0 \left( \frac{\sin^2 \beta}{\beta^2} \right) \]
\[ \beta = \frac{1}{2} kb \sin \theta \]

\[ I = 4I_0 \left( \frac{\sin^2 \theta}{\beta^2} \right) \cos^2 \alpha, \quad \beta = \frac{1}{2} kb \sin \theta, \quad \alpha = \frac{1}{2} ka \sin \theta \]

\[ I = I_0 \left( \frac{\sin^2 \theta}{\beta^2} \right) \left( \frac{\sin^2 \alpha}{\sin^2 \alpha} \right), \quad \beta = \frac{1}{2} kb \sin \theta, \quad \alpha = \frac{1}{2} ka \sin \theta \]

\[ l_t = c r_0, \quad l_t = \frac{c}{\Delta f}, \quad l_\lambda = \frac{\lambda^2}{\Delta \lambda}, \quad \Delta \lambda = \frac{\lambda^2}{l} \]

\[ e^\alpha = \cos \alpha + i \sin \alpha, \quad \sin^2 \theta + \cos^2 \theta = 1, \quad \sin^2 \theta = \frac{1 - \cos 2\theta}{2}, \quad \cos^2 \theta = \frac{1 + \cos 2\theta}{2} \]

\[ (1 - x)^n = 1 - Nx \quad \text{(if } |x| < 1) \]
1. 1a) One converging lens and one diverging lens with focal lengths of 10 cm are placed coaxially 60 cm apart. Find the position of the image of an object placed 15 cm to the left of the first lens (converging lens). Find out the location and the magnification of the final image relative to the original object. Carefully sketch a ray-tracing diagram showing the object, the intermediate image, the final image, and ray refraction through each lens. [10%]

1b) A glass hemisphere is coated with silver to be reflective over its curved surface, as shown in the figure below. A small air bubble in the glass is located on the central axis through the hemisphere 5 cm from the plane surface. The radius of curvature of the spherical surface is 7.5 cm, and the glass has an index of 1.50. Looking along the axis into the plane surface, one sees two images of the bubble. Carefully sketch a ray-tracing diagram and find out the locations of the images. [10%]
2. 2a) Let a collimated beam of white light fall on one refracting face of a prism and let the light emerging from the second face be focused by a lens onto a screen. Suppose that the linear dispersion at the screen is 2.0 nm/mm. By introducing a narrow "exit slit" in the screen, one has a type of monochromator that provides a nearly monochromatic beam of light. Sketch the setup. For an exit slit of 0.02 cm, what is the coherence time and coherence length of the light of mean wavelength 500 nm? [10%]

2b) Explain the Brewster's condition and its physical implication. [10%]
3. In order to successfully use optical fibres \((n_1 > n_2)\), one must couple light into the fibres. There are a few methods to accomplish this task.

3a) The simplest method is shown below. Determine the acceptance angle (or maximum entrance cone angle). [6%]

![Diagram of optical fibre](image)

3b) Would the method shown below be able to couple light into the optical fibre? If so, determine the acceptance angle. If this method will not work, explain why not. [6%]

![Diagram of optical fibre](image)

3c) If the spreads in arrival times \((\delta t)\) of pulses from a laser diode (LD) and a light-emitting diode (LED) are 0.11 ns and 2.2 ns, respectively, determine the corresponding maximum frequencies that can be transmitted in the fibre. Explain the reason causing the difference between the LD and LED. [8%]
4. a) Monochromatic light of wavelength $\lambda = 600$ nm is normally incident on a soap film ($n_f = 1.40$) immersed in air. What is the minimum thickness of the soap film if constructive interference occurs in the reflected light? [8%]

4b) Two infinitely small slits are located as shown below. A plane wave with vacuum wavelength ($\lambda$) traveling in the +x direction and amplitude $A_0$ strikes the slits. The medium to the right of the slits, $x>0$, has a refractive index $n = n(x)$. In this problem, consider the interference effect only (ignore any refraction and diffraction effects at the slits) and answer the following questions:

i. Using the approximation $d >> y$ (i.e., $\theta << 1$). Find an expression for the interference pattern at $x = d$ for $n(x) = n_0$ (in other words, $n(x)$ is a constant). [6%]

ii. Using the approximation $d >> y$ (i.e., $\theta << 1$). Find an expression for the interference pattern at $x = d$ for $n(x) = n_0 + x\eta$, where $\eta$ is a constant. [6%]
5. The figure below shows the variation of irradiance with vertical distance from the axis at the screen, which is the Fraunhofer diffraction pattern measured from N identical parallel slits. Suppose each slit has a slit width $b$ with slit separation $a$. The distance from the slits to the screen is $f$. The wavelength of the light to pass through the slits is $\lambda$.

5a) How many slits are in the array? [4%]

5b) Write the expression for the envelope (i.e. the dashed line) in the figure and explain its physical implication. [4%]

5c) Using the parameters given above, find out expressions for $D$ and $X$, as indicated in the figure. [6%]

5d) Demonstrate that, $I_1 \approx I_0/9$, if $a >> b$. [6%]
6. 6a) Consider the following Mach-Zehnder interferometer:

There are two beamsplitters (BS) that split an incident field equally into two parts. If the input intensity is \( I_i \), show that the output intensity \( I_o \) is \( I_o = I_i \cos^2(\phi/2) \), where \( \phi \) is the relative phase difference between beams that travel branches 1 and 2. [10%]

6b) In order to rotate the polarization direction of a linearly polarized light beam by 90° and obtain an output irradiance of 95% of the original value, several ideal identical polarizers will be used. How many polarizers will be needed? [10%]