1. (a) Write down the system of equations that represent an inertial balance? 2 pt.
   (b) What are the characteristics of an ocean flow in inertial balance? 3 pt.

2. (a) Write down the system of equations that represent an Ekman balance? 2 pt.
   (b) What boundary condition at the sea surface was imposed by Ekman to solve these equations? 1 pt.
   (c) What are the characteristics of the solution of this system of equations? Support your answer with a clear sketch. 4 pt.
   (d) Explain what the Ekman transport is and how it relates to coastal upwelling. Support your answer with a sketch. 2 pt.

3. Show that for a homogeneous ocean (i.e. constant density $\rho_0$) in hydrostatic equilibrium, the geostrophic balance
   
   $$fv = \frac{1}{\rho_0} \frac{\partial p}{\partial x},$$

   can be expressed as
   
   $$fv = g \frac{\partial \eta}{\partial x},$$

   where $\eta$ is the sea-surface elevation (other variables follow the standard notation introduced in class). 4 pt.

4. Using the information provided in Figure 1, estimate as best as you can the geostrophic current (intensity and direction) at the location of the $\times$ sign in the North Atlantic. 4 pt.

5. Figure 2 shows a 7 days prediction of the water level in St. John's. Explain qualitatively (i.e. no equations needed) the features of this tidal signal. In particular explain:
   
   (a) Why are there two tides per day while the Earth with respect to the Moon and the Sun makes only one revolution per day; 2 pt.
   (b) Why two consecutive tides have quite different amplitudes; 2 pt.
   (c) Why the average tidal range (i.e. the difference between high and low tide) decreases with time over the prediction period. 2 pt.

   Support your answer with a clear sketch of the Earth-Moon-Sun system and show with vectors the forces involved.
6. Assume that the density profile in some region of the ocean can be represented by the relation

\[ \rho(z) = \rho_0 - \gamma z, \]

where \( \rho_0 = 1023 \text{ kg m}^{-3} \) and \( \gamma = 0.01 \text{ kg m}^{-4} \).

(a) What is the buoyancy frequency at \( z = -100 \text{ m} \)? Show your work.  
(b) What is the pressure at \( z = -100 \text{ m} \)? Show your work.  

7. Figure 3 shows an internal wavetrain propagating at the interface of a two layer system in the St. Lawrence Estuary. The densities of the top layer and bottom layers are, respectively, \( \rho_1 = 1020 \text{ kg m}^{-3} \) and \( \rho_2 = 1025 \text{ kg m}^{-3} \).

(a) Given the information on the figure would you consider the internal waves seen between 900 m and 1100 m from the shore to be “shallow water” or “deep water” waves? Justify your answer.  
(b) Estimate the phase speed of the wave at 1000 m from the shore.  

![Four-Year Mean Sea-Surface Topography (cm)](image)

Figure 1: Global distribution of the sea surface height.
Figure 2: 7 days prediction of the water level in St. John’s.

Figure 3: An internal wavetrain running into a sloping boundary in the St. Lawrence Estuary. The waves propagate from right to left. These observations were collected in 2004 by Marina for her Ph.D. research.
Equation Sheet

\[ \frac{\partial u}{\partial t} + \frac{u}{\partial x} \frac{\partial u}{\partial x} + \frac{v}{\partial y} \frac{\partial u}{\partial y} + \frac{w}{\partial z} \frac{\partial u}{\partial z} - f v = -\frac{1}{\rho_0} \frac{\partial p}{\partial x} + A_z \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) + A_z \frac{\partial^2 u}{\partial z^2} \]

\[ \frac{\partial v}{\partial t} + \frac{u}{\partial x} \frac{\partial v}{\partial x} + \frac{v}{\partial y} \frac{\partial v}{\partial y} + \frac{w}{\partial z} \frac{\partial v}{\partial z} + f u = -\frac{1}{\rho_0} \frac{\partial p}{\partial y} + A_z \left( \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right) + A_z \frac{\partial^2 v}{\partial z^2} \]

\[ 0 = -\frac{1}{\rho} \frac{\partial p}{\partial z} - g \]

\[ \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \]

\[ f = 2\Omega \sin \phi \]

\[ \Omega = \frac{2\pi}{1 \text{ day}} = \frac{2\pi}{86400 \text{ s}} = 7.2722 \times 10^{-5} \text{ rad s}^{-1} \]

\[ N^2 = -\frac{g}{\rho_0} \frac{\partial \rho}{\partial z}, \quad g' = \frac{\Delta \rho}{\rho_0} \]

\[ c = \sqrt{\frac{g}{k} \tanh(kH)}, \quad c = \sqrt{g' H_e}, \quad H_e = \frac{h_1 h_2}{h_1 + h_2}, \quad c = \sqrt{\frac{g}{k} \frac{\Delta \rho}{(\rho_2 + \rho_1)}} \]

\[ w_E = \frac{1}{\rho_0 f} \left( \frac{\partial \tau_y}{\partial x} - \frac{\partial \tau_x}{\partial y} \right) \]

\[ r = V/\|f\|, \quad T = 2\pi/\|f\| \]

\[ R = \sqrt{gH/\|f\|} \]

\[ D_E = \pi \sqrt{\frac{2A_z}{\|f\|}} \]

\[ g = 9.81 \text{ m s}^{-2} \]

1° of latitude = 111 km