MEMORIAL UNIVERSITY OF NEWFOUNDLAND
Department of Physics and Physical Oceanography
Physics 3230 (Classical Mechanics II)
Final Examination
Friday April 13, 2007, 3:00-5:00 p.m.

Answer all 5 questions

Formulae

\[ r = \frac{c}{1 + \varepsilon \cos \phi} \]
\[ c = \frac{l^2}{\gamma \mu} \]
\[ \gamma = \frac{GmM}{\mu} \quad \mu = \frac{mM}{m + M} \]
\[ g = \frac{GM}{R^2} \]

\[ \mathbf{v} = \mathbf{v}_0 + \mathbf{v}' + \omega \times \mathbf{r}' \]
\[ \mathbf{a} = a_0 + \mathbf{a}' + 2\omega \times \mathbf{v}' + \omega \times (\omega \times \mathbf{r}') + \alpha \times \mathbf{r}' \]

\[ \{H\} = [I] \{\omega\} \]
\[ p_k = \frac{\partial L}{\partial q_k} \]
\[ H = \sum_{i=1}^{n} p_i q_i - L \]
\[ q_k = \frac{\partial H}{\partial p_k} \quad (k = 1, \ldots n) \]
\[ p_k = \frac{\partial H}{\partial q_k} \quad (k = 1, \ldots, n) \]
Qu. 1.
(a) [10 marks] A spacecraft is describing an elliptic orbit around the earth. The orbit has minimum radius (measured from the earth’s centre) \( r_A \) at point \( A \) and maximum radius \( r_B \) at point \( B \). Assuming that the mass \( m \) of the spacecraft is much smaller than the mass \( M \) of the earth, show that

\[
\frac{1}{r_A} + \frac{1}{r_B} = \frac{2gR^2}{h^2}
\]

where \( g \) is the acceleration due to gravity at the earth’s surface, \( R \) is the radius of the earth and \( h \) is the angular momentum of the spacecraft per unit mass, i.e. \( h = \ell/m \) where \( \ell \) = angular momentum of spacecraft about earth’s centre.

[b] [10 marks] The minimum altitude of the spacecraft above the earth’s surface is \( h_A = 2640 \text{ km} \) and the maximum altitude is \( h_B = 10560 \text{ km} \). The earth’s radius is \( R = 6370 \text{ km} \). Determine the speed of the spacecraft at \( A \) and \( B \).

Qu. 2 [20 marks]
An object of mass \( m = 1500 \text{ kg} \) is observed to be at a height of 5000 km vertically above a point \( A \) on the earth’s surface at colatitude \( \theta = 30^\circ \). Relative to \( A \), the object has a constant velocity of 8000 \( m/s \) due south (direction \(-\vec{j}'\)). Determine the resultant force on the object in terms of unit vectors at \( A \): \( \vec{i}' \) pointing east, \( \vec{j}' \) pointing north and \( \vec{k}' \) pointing vertically upwards. The earth’s angular speed is \( \omega = 7.3 \times 10^{-5} \text{ rad/s} \) and its radius is \( R = 6370 \text{ km} \).

Qu. 3 [20 marks]
Two uniform rods \( CD \) and \( DE \) each of mass \( m \) and length \( a \) are welded to shaft \( AB \) as shown to form a straight rod \( CDE \) of mass \( 2m \) and length \( 2a \). At the instant shown, both \( CDE \) and \( AB \) lie in the \( x - y \) plane and the structure rotates around \( AB \) with angular velocity \( \omega = \omega_i \). Determine the angular momentum of \( CDE \) about \( D \) in the \( DXYZ \) frame.
Qu. 4 [20 marks]

AB, BC and CD are identical springs of negligible mass and stiffness $k$. The masses $m$ fixed to the springs at B and C are displaced by small distances $x_1$ and $x_2$ from their equilibrium positions along the line of the springs. Show that the system has natural frequencies $\omega_1 = \sqrt{k/m}$ and $\omega_2 = \sqrt{3k/m}$ and find the normal modes. It is not necessary to find the normal coordinates.

Qu. 5 [20 marks]

Two particles $A$ and $B$ of masses $m$ and $M$ respectively are connected by a light inextensible string of length $a$ which passes through a smooth hole $O$ in a smooth horizontal table. Particle $B$ is suspended below the table and particle $A$ rests on the table. Particle $A$ has two degrees of freedom $r, \theta$ in the plane of the table. Using generalised coordinates $q_1 = r$ and $q_2 = \theta$ as shown, derive Hamilton's equations of motion and show that the radial acceleration of $A$ is inversely proportional to the cube of its distance from $O$. 