MEMORIAL UNIVERSITY OF NEWFOUNDLAND
Department of Physics and Physical Oceanography
Physics 3230 (Classical Mechanics II)
Final Examination
April 30, 2006, 3:00-5:00 p.m.

Answer all 5 questions

Formulae

\[ r = \frac{c}{1 + \varepsilon \cos \phi} \]
\[ c = \frac{l^2}{\gamma \mu} \]
\[ \gamma = \frac{G m M}{\mu} \quad \mu = \frac{m M}{m + M} \]
\[ g = \frac{G M}{R^2} \]

\[ [K] = [M]^{-\frac{1}{2}} [K][M]^{-\frac{1}{2}} \quad [S] = [M]^{-\frac{1}{2}} [P] \quad \{ r \} = [S]^{-1} \{ x \} \]

\[ v = v_{O'x} + v' + \omega \times r' \]
\[ a = a_{O'} + a' + 2\omega \times v' + \omega \times (\omega \times r') + \alpha \times r' \]

\[ \{ H \} = [I] \{ \omega \} \]
\[ p_k = \frac{\partial L}{\partial \dot{q}_k} \]
\[ H = \sum_{i=1}^{n} p_i \dot{q}_i - L \]
\[ \dot{q}_k = \frac{\partial H}{\partial p_k} \quad (k = 1, \ldots n) \]
\[ \dot{p}_k = -\frac{\partial H}{\partial q_k} \quad (k = 1, \ldots, n) \]

Disk of mass \( m \), radius \( r \)

\[ I_{xx} = I_{yy} = \frac{1}{2} m r^2 \]
\[ I_{zz} = \frac{1}{2} m r^2 \]
1. A space shuttle is to rendezvous with an orbiting laboratory \( A \) which circles the earth (centre \( O \), mass \( M \), radius \( R \)) at a constant altitude of 360 km. The shuttle \( B \) has reached an altitude of 60 km when its engine is shut off, and its velocity \( \vec{v}_B \) forms an angle \( \phi_0 = 50^\circ \) with the vertical \( OB \) at that time. \( R = 6370 \text{ km} \).

![Diagram of space shuttle and laboratory](image)

(a) [5 marks] Using conservation of energy, show that

\[
v_A^2 = v_B^2 - \frac{2GM}{r_B} \left( 1 - \frac{r_B}{r_A} \right)
\]

where \( r_A = OA \), \( r_B = OB \) and the speeds at \( B, A \) are \( v_B, v_A \) respectively. \( G \) is the gravitational constant.

(b) [15 marks] What should be the value of \( v_B \) if the shuttle’s trajectory is to be tangent at \( A \) to the orbit of the trajectory?

2. [20 marks] The mechanism shown is used to raise a worker to the elevation of overhead electric and telephone wires. The entire mechanism rotates at the constant rate \( \omega_1 = 0.15 \text{ rad/s} \) about the \( y' \) axis. The frame \( O'x'y'z' \) is attached to the mechanism and rotates with it. The angle between arm \( O'B \) and the horizontal is constant, while arm \( BC \) is being lowered at the constant rate \( \omega_2 = 0.20 \text{ rad/s} \). The arms \( O'B \) and \( BC \) are each 4.5 m long. Determine the acceleration of point \( C \) at the instant shown in terms of the unit vectors of the \( O'x'y'z' \) frame.

![Diagram of worker and mechanism](image)
3. [20 marks] A thin homogeneous disk of mass \( m \) and radius \( r \) is rigidly mounted on the horizontal axle \( AB \). The plane of the disk forms an angle \( \beta \) with the vertical and the axle rotates with angular speed \( \omega \) about the \( x \) axis. Determine the angular momentum of the disk about its centre in the \( x-y-z \) coordinate frame.

4. [20 marks] A light elastic spring of stiffness \( k \) is clamped at its upper end and supports a particle of mass \( m \) at its lower end. A second spring of stiffness \( k \) is fastened to the particle and, in turn, supports a particle of mass \( 2m \) at its lower end. The equations of motion are

\[
\begin{align*}
mx_1 + 2kx_1 - kx_2 &= 0 \\
2mx_2 - kx_1 + kx_2 &= 0
\end{align*}
\]

where \( x_1, x_2 \) are the displacements of the masses from the static equilibrium positions. Determine the natural frequencies \( \omega_k \), normal mode vectors \( \{s\}_k \) and normal coordinates \( r_k \) such that the equations of motion may be written in the de-coupled form

\[
\ddot{r}_k + \omega_k^2 r_k = 0 \quad (k = 1, 2)
\]

5. [20 marks] A particle \( P \) of mass \( m \) is fixed to the centre of a weightless rod of length \( a \) and the ends of the rod are constrained to move along smooth walls as shown. Using generalised coordinate \( \theta \) as shown, determine Hamilton’s equations of motion and find the rod’s angular acceleration when \( \theta = 30^\circ \).