INSTRUCTIONS:

1. Do all **SIX (6) questions** in section 1 and all **THREE (3) questions** in section 2. Do **TWO questions** from section 3. Marks are indicated in the left margin. Section 1 contains 36 marks, Section 2 contains 36 marks, and section 3 contains 28 marks. Budget time accordingly.

2. You may use a calculator. All other aids are prohibited.

3. Write answers neatly in space provided. If necessary, continue onto the back of the page.

4. Do not erase or use “whiteout”. Draw a line neatly through material to be replaced.

5. Assume all information given is accurate to 3 significant figures.

6. Don’t panic. If something isn’t clear, **ASK**!

**SEE LAST PAGE FOR SOME POTENTIALLY USEFUL FORMULAE AND CONSTANTS**

For office use only:
SECTION 1: There are 6 SHORT ANSWER questions. Each is worth 6 marks for a maximum of 36 marks. Budget time accordingly.

[6] 1. A simple pendulum has a mass of 0.25 kg and a length of 0.75 m. It is displaced through an angle \( \theta = 15^\circ \) (\( \theta = 0.262 \) radians) and then released.

(a) What is the maximum tangential speed?

(b) Obtain an expression for the angular acceleration, \( \alpha = d^2\theta / dt^2 \), when the pendulum is at an arbitrary angle \( \theta \), in radians, with respect to the vertical.

[6] 2. A 200 \( \mu \)C charge is at the centre of an uncharged (3-dimensional) insulating cubic shell with 50 cm edges. This shell, in turn, is centred inside of a metal (conducting) cubic shell with 100 cm edges.

(a) Find the electric flux through one face of the insulating cubic shell.

(b) What is the total surface charge spread over the entire inner surface of the conducting cubic shell?

(c) The charge is moved slightly to the left so it remains within the insulating cubic shell but is no longer at the centre. Will this change your answers for (a), (b), both, or neither. (Hint: No calculations are needed for this part.)
[6] 3. (a) A wave is described by the function \( y(x,t) = 0.017 \text{ m} \cdot \sin\left[8.4 \text{ m}^{-1} \cdot x - \left(75 \text{ s}^{-1}\right) \cdot t\right] \). At what speed does this wave travel?

(b) Waves with a wavelength of 638 nm are incident on a barrier with two small slits separated by a distance \( d = 5 \times 10^{-6} \text{ m} \). The two parallel rays point toward a spot on a distant screen at which the waves from the two slits interfere constructively. If that is the first bright spot above the centre of the interference pattern on the screen, what is the angle \( \theta \)? You can assume that \( \theta \) is small.

[6] 4. The diagrams show two arrangements of polarizers with transmission axes oriented in different directions in the \( y-z \) plane. In each case, unpolarized light traveling in the \( x \) direction first passes through a polarizer with its transmission axis parallel to the \( z \) direction and emerges with an intensity \( I_0 \). Subsequent polarizers are labeled by the angle between their transmission axes and the \( z \) direction. For each arrangement, calculate the intensity, in terms of \( I_0 \), of the light emerging from the last polarizer.
[6] 5. A square loop of wire is situated in a region of uniform magnetic field with its diagonal parallel to the magnetic field lines as shown. Each side of the square is 1.7 m long. The magnetic field is \( B = 2.5 \, \text{T} \, \hat{j} \).

(a) What is the magnetic force on the side \( ab \)? Give your answer in unit vector notation.

(b) What is the magnetic moment, \( \vec{\mu} \), of the loop? Give your answer in unit vector notation.

(c) What is the torque, \( \vec{\tau} \), on the loop? Give your answer in unit vector notation.

[6] 6. Two glass plates (\( n = 1.45 \)) are touching along one edge and separated by a thin wire along the other as shown. When they are illuminated by monochromatic light, alternating dark and bright bands are seen in reflection.

(a) Briefly explain why there is a dark band, which we will call the 1st dark band, at the end where the plates are touching.

(b) The next dark band (the 2nd) occurs where the plates are separated by 319 nm. What is the wavelength of the light?

(c) Imagine that the space between the plates is now filled with light oil having an index of refraction \( n = 1.33 \). Does the spacing between adjacent bands increase or decrease? Briefly justify your answer.
SECTION 2: Do ALL three (3) questions. Each question is worth 12 marks for a maximum of 36 marks.

[12] 7. One point charge, \( q_1 = -3.0 \times 10^{-9} \text{ C} \), is located 15 cm from the origin along the y axis. A second point charge, \( q_2 = +5.0 \times 10^{-9} \text{ C} \), is located 15 cm from the origin along the x-axis as shown.

(a) What is the force on charge 1 due to charge 2? Give your answer in unit vector notation.

(b) What is the magnitude of the electric field at the origin (point P)?

(c) What is the electric potential at the origin (point P)?

(d) How much work would be done by the electric field if charge 2 were moved from the position shown to the origin? (Be sure to indicate the sign.)
8. A student who likes to do things the hard way wishes to measure the length, \( L_T \), of a tube that is open at one end and closed at the other. He has a string with a length \( L_S = 0.4 \) m and a mass of 0.006 kg. He finds that the fundamental resonance of the string matches one harmonic of the tube (not the fundamental) when the tension in the string is 576 N. To match the next harmonic of the tube, he must increase the string tension to 1600 N.

(a) What is the fundamental frequency of the string when its tension is 576 N?

(b) What is the fundamental frequency of the string when its tension is 1600 N?

(c) Based on your previous answers, what is the fundamental frequency of the tube? (Hint: For a tube closed at one end, how is the difference in frequency between one harmonic and the next related to the fundamental frequency?)

(d) What is the length of the tube? Assume that the speed of sound in air is 343 m/s. (Hint: How is the length of the tube related to the wavelength of the fundamental tube harmonic?)
[12] 9. Two very large square metal sheets with are separated by 5.0 mm. There is a uniform electric field in the space between them and the potential difference between them is 1500 V.

(a) What is the magnitude of the electric field between the plates?

(b) An ion with a charge of \( +3.2 \times 10^{-19} \) C and a mass of \( 6.6 \times 10^{-27} \) kg is released from rest on the surface of the positive plate. What is its speed as it passes through a small hole on the negative plate?

(c) After it emerges from the hole in the negative plate, the ion enters a region of space containing a 9.4 T magnetic field oriented perpendicular to its velocity. What is the magnitude of the magnetic force on the ion?

(d) What is the radius of the ion’s orbit while it is in the region of magnetic field?

(e) From the direction in which the particle is deflected, do you infer that the magnetic field points into the page or out of the page?
SECTION 3: Do TWO (2) of the three questions. Each question is worth 14 marks for a total of 28 marks. Indicate clearly the one question that you do not want marked by drawing a line through it (don’t erase!).

[14] 10. A segment (arc) of a non-conducting charged ring, centred on the origin, subtends the angle from \( \theta_1 = -60^\circ \) to \( \theta_2 = +60^\circ \) as shown. The radius of the ring is 0.45 m and the linear charge density on the ring is \( \lambda = 7.0 \times 10^{-12} \text{ C/m} \).

(a) Without doing an integral, what is the \( y \)-component, \( E_y \), of the total electric field at the origin due to the entire charged ring segment (from \( \theta_1 = -60^\circ \) to \( \theta_2 = +60^\circ \))?  

(b) What is the contribution, \( dE_x \), to the \( x \) component of the electric field at the origin due to the charge \( dq \) in a segment \( ds \) at angle \( \theta \)? (Hint: think about how to write \( dq \) in terms of the radius of the segment and the angle, \( d\theta \) subtended by the segment \( ds \).)  

(c) What is the \( x \)-component, \( E_x \), of the total electric field at the origin due to the entire charged ring segment (from \( \theta_1 = -60^\circ \) to \( \theta_2 = +60^\circ \)). (Integrals are provided on page 11.)
11. Two sliding metal bars of length \( l = 15.0 \, \text{cm} \) are moving along two parallel rails, in opposite directions, with constant speeds of \( v = 0.6 \, \text{m/s} \), as shown in the figure below. The rails are located in a uniform magnetic field with a magnitude 0.47 T that is directed into the page as shown.

(a) Calculate the rate of change of the magnetic flux within the loop formed by the sliders and the rails.

(b) The total resistance in the circuit is 25.0 \( \Omega \). What is the magnitude of the current flowing around the circuit?

(c) In what direction does the current flow? Indicate this clearly on the diagram and briefly justify your answer.

(d) What force (magnitude and direction) must be applied to the right slider to keep it moving at constant speed?
[14] 12. Two infinitely long wires run parallel to the z-axis (which comes out of the page in the picture). Wire #1 carries a 1.5 A current into the page at \( x = -0.03 \) m, \( y = 0 \) m and wire #2 carries a 1.5 A current out of the page at \( x = +0.03 \) m, \( y = 0 \) m as shown.

(a) On the diagram, draw and label the contributions from wire #1 and wire #2 to the magnetic field at point \( P \) which is located at \( x = 0 \) cm, \( y = 0.03 \) m as shown.

(b) Calculate the total magnetic field at point \( P \). Express your answer in unit vector notation.

(c) Imagine a particle of charge \( q = +3.2 \times 10^{-19} \) C along the z direction through point \( P \) with a velocity \( \vec{v} = 5 \times 10^5 \) m/s \( \hat{k} \). What is the force on this particle? Express your answer in unit vector notation.

(d) What is the force per unit length exerted on wire #2 due to the magnetic field from wire #1? Express your answer in unit vector notation.
Some Potentially Useful Formulae:

\[ \frac{d^2x}{dt^2} = -\omega^2 x \]

\[ F_{12} = k \cdot \frac{q_1 q_2}{r_{12}^2} \]

\[ U_{12} = k \cdot \frac{q_1 q_2}{r_{12}} \]

\[ F = \frac{\mu_0 I_1 I_2}{2\pi} \]

\[ \oint B \cdot d\mathbf{s} = \mu_0 I \]

\[ \Delta U = -\int \mathbf{E} \cdot d\mathbf{s} \]

\[ \Phi = \oint \mathbf{B} \cdot d\mathbf{A} \]

\[ \Delta U = q\Delta V \]

\[ c = -N \frac{d\Phi}{dt} \]

\[ \Delta \phi = \frac{2\pi}{\lambda} \delta \]

\[ d\sin \theta_{\text{right}} = m\lambda \quad (m = 0, 1, 2 \ldots) \]

Equations of Electromagnetism (Maxwell’s Equations):

\[ \oint \mathbf{E} \cdot d\mathbf{A} = \frac{\partial \Phi}{\partial t} \]

\[ \oint \mathbf{B} \cdot d\mathbf{A} = 0 \]

\[ \oint \mathbf{B} \cdot d\mathbf{A} = \mu_0 I + \mu_0 \varepsilon_0 \frac{d\Phi}{dt} \]

Physical constants:

\[ k_e = \frac{1}{4\pi \varepsilon_0} = 8.99 \times 10^9 \text{ N} \cdot \text{m}^2 / \text{C}^2 \]

\[ \mu_0 = 4\pi \times 10^{-7} \text{ T} \cdot \text{m} / \text{A} \]

\[ m_e = 9.11 \times 10^{-31} \text{ kg} \]

\[ \varepsilon_0 = 8.85 \times 10^{-12} \text{ C}^2 / \text{N} \cdot \text{m}^2 \]

\[ g = 9.81 \text{ m/s}^2 \]

Mathematical formulae

\[ \mathbf{A} \cdot \mathbf{B} = AB \cos \theta = A_i B_j + A_j B_i + A_i B_j \]

\[ \mathbf{A} \times \mathbf{B} = AB \sin \theta \]

\[ i \times j = k \quad j \times k = i \quad k \times i = j \]

\[ \mathbf{A} \times (\mathbf{B} - (\mathbf{A} \cdot \mathbf{B}) \mathbf{i}) + (\mathbf{A} \cdot \mathbf{B}) \mathbf{i}] = (\mathbf{A} \cdot \mathbf{B}) \mathbf{i} + (\mathbf{A} \cdot \mathbf{B}) \mathbf{i} + (\mathbf{A} \cdot \mathbf{B}) \mathbf{i} \]

\[ \int \frac{dx}{\sqrt{x^2 + a^2}} = \ln(x + \sqrt{x^2 + a^2}) \]

\[ \int \frac{dx}{\sqrt{x^2 + a^2}} = -\frac{1}{\sqrt{x^2 + a^2}} \]

\[ \int \frac{dx}{\sqrt{x^2 + a^2}} = \frac{1}{a} \tan^{-1} \left( \frac{x}{a} \right) \]

\[ V_{\text{sphere}} = \frac{4}{3} \pi r^3 \quad A_{\text{sphere}} = 4\pi r^2 \]

\[ V(x) = \int_{x_0}^{x} dx = \ln x \]

\[ \int \sin ax \, dx = -a^{-1} \cos ax \]

\[ \int \cos ax \, dx = a^{-1} \sin ax \]