MEMORIAL UNIVERSITY OF NEWFOUNDLAND
DEPARTMENT OF PHYSICS AND PHYSICAL OCEANOGRAPHY
Physics 1050

FINAL EXAMINATION
December 7, 2007

INSTRUCTIONS:

1. THIS EXAMINATION PAPER CONSISTS OF 8 QUESTIONS.

2. DO ALL 8 QUESTIONS. ALL QUESTIONS CARRY EQUAL MARKS.

3. Write your name and student number on each page.

4. Feel free to remove the formulae sheet located on the back of the examination.

5. You may use a calculator. All other aids are prohibited.

6. Use only the paper provided. No other books, notes or papers are permitted. The examination paper must not be taken from the room.

7. Write answers neatly in space provided. If necessary, continue onto the back of the page.

8. Do not erase or use “whiteout” to correct answers. Draw a line neatly through material to be replaced and continue with correction.

9. Don’t panic. If something isn’t clear, ASK!
1. (a) Potential energy as a function of position $x$ for a block-spring system is shown below. The positive $x$ direction is to the right as indicated.

(i) Identify any equilibrium position(s) for the block and indicate whether each is a point of stable, unstable or neutral equilibrium.

(ii) Determine whether the force is negative, positive or zero at points A, B and C as indicated on the diagram.

(iii) At which of the three points (A, B or C) is the magnitude of the force the largest? Give a reason based on the energy diagram.

(b) A 30.0 kg child runs with a speed of 2.50 m/s tangential to the rim of a stationary merry-go-round as shown in the figure. The merry-go-round has a radius of 2.60 m and a moment of inertia of $5.00 \times 10^2$ kg m$^2$.

(i) What is the angular momentum of the child with respect to the centre of the merry-go-round?

(ii) What is the moment of inertia of the child about the merry-go-round’s axis of rotation?

(iii) Determine the angular speed of the system consisting of child and merry-go-round after the child jumps on and it begins to rotate.
2. A 5.0 kg mass is placed on a frictionless ledge and is pushed against a spring that is compressed from its equilibrium length by 0.070 m (see figure below). The spring constant of the spring is 850.0 N/m. The spring obeys Hooke's Law. The edge of the ledge is 1.5 m above the ground. The spring is released and the mass is projected horizontally.

(a) How much elastic potential energy is stored in the compressed spring?

(b) What is the speed of the mass in m/s after it looses contact with the spring?

(c) How much work in total does the spring do on the mass after the spring is released?

(d) How long does it take for the mass to reach the ground after it reaches the edge of the ledge?

(e) How far does the mass travel horizontally after it reaches the edge of the ledge?

(f) Using unit vector notation, what is the velocity of the mass just before it lands on the ground?
3. A van (m = 1500.0 kg) is rounding a curve on a road banked at an angle $\theta = 30.0^\circ$. The van is going around at a constant speed in a circle of radius $r = 30.0 \text{ m}$. The coefficient of static friction between the road surface and the tires of the van is 0.350. Assume that the van is about to slip down the incline so that the static frictional force takes on its maximum possible value. (Hint: Take the frictional force as directed up the incline.)

(a) Draw a free body diagram for the car labeling all the external forces acting on it.

(b) What is the magnitude of the normal force acting on the car?

(c) What is the speed of the car?
4. Two blocks of mass \( m_1 = 25.0 \, \text{kg} \) and \( m_2 = 50.0 \, \text{kg} \) are connected by a string of negligible mass (see figure below). The pulley is frictionless and also of negligible mass. The coefficient of kinetic friction between the 25.0 kg block and surface is 0.250. The string connected to \( m_1 \) makes an angle of 35.0° with the horizontal as shown.

(a) Draw a free body diagram for the two blocks labeling all the external forces acting on them.

(b) Determine the tension in the string.

(c) Determine the acceleration of the system.

(d) The massless frictionless pulley is now replaced with a real pulley with a moment of inertia of \( 5.00 \times 10^{-2} \, \text{kg} \cdot \text{m}^2 \) and a radius of 0.100 m. By applying the rotational analog of Newton’s second law, write down the equation for the net torque on the pulley and determine the magnitude of the angular acceleration. The tensions in the strings attached to \( m_1 \) and \( m_2 \) are \( T_1 = 87.0 \, \text{N} \) and \( T_2 = 227 \, \text{N} \), respectively.
5. A bumper car moving at 10.0 m/s to the right (the positive x direction) strikes another stationary bumper car of the same mass. After the collision, the first car moves at 7.50 m/s at an angle of 60.0° with respect to the original line of motion (the cars do not stick together after the collision). The mass of each car is 100.0 kg.

(a) What is the velocity of the second car (its magnitude and direction with respect to the original line of motion) immediately after the collision?

(b) Express the final velocities of the two cars in unit vector notation.

(c) Calculate the impulse experienced by the second car during the collision.

(d) Show that the velocity of the center of mass of the two cars is the same before and after the collision.
6. A uniform ladder of length \( L = 5.00 \text{ m} \) and mass \( m_L = 15.0 \text{ kg} \) rests against a frictionless wall. The ladder makes an angle \( \theta = 60.0^\circ \) with the horizontal. A firefighter of mass \( m_F = 70.0 \text{ kg} \) is a distance of 4.00 m from the bottom of the ladder as shown. There is friction between the ground and the bottom of the ladder.

(a) Draw a free body diagram showing all external forces acting on the ladder.

(b) Determine the magnitude of the force exerted by the wall on the ladder.

(c) Determine the horizontal and vertical components of the force that the ground exerts on the base of the ladder.
7. A 3.2 kg uniform rod of length 1.6 m is attached to a frictionless pivot 0.40 m from its centre of mass as shown. The rod is initially held at an angle of 50.0° from the vertical.

(a) The moment of inertia for a thin rod of length L and mass M about an axis through its centre of mass is \( I = \frac{1}{12} ML^2 \). Use the parallel axis theorem to find the moment of inertia for this rod about the pivot point shown.

(b) If the rod is released from rest at the position shown,
   i. Show that when the rod is horizontal its rotational kinetic energy is equal to 8.1 J.
   ii. What is the angular speed of the rod when it is horizontal?
   iii. What is the speed of the centre of mass when the rod is horizontal?

(c) By using the torque exerted on the rod by gravity, find the angular acceleration of the rod at the instant it is horizontal.
8. A 1.50 kg object is attached to a spring and placed on a frictionless horizontal surface. A horizontal force of 15.0 N is required to hold the object at rest when it is pulled 0.0200 m from its equilibrium position (the origin of the x axis). The object is now released from rest with an initial position of \( x_i = 0.0200 \) m, and it subsequently undergoes simple harmonic oscillations.

(a) Determine the force constant of the spring.
(b) What is the angular frequency of the oscillations?
(c) What is the amplitude of the oscillations?
(d) Determine the maximum speed of the object. At what value of \( x \) does this occur?
(e) Determine the magnitude of the maximum acceleration of the object. At what value(s) of \( x \) does this occur?
(f) What is the total energy of the oscillating system?
(g) Determine the earliest time at which the object’s position is equal to one third of the maximum value.
(h) Determine the speed of the object when the object’s position is equal to one third of the maximum value.
FORMULAE SHEET

For constant $a$:
\[
\begin{align*}
  x &= x_0 + v_0 t + \frac{1}{2} a t^2 \\
  v^2 &= v_0^2 + 2a(x - x_0) \\
  v &= v_0 + at \\
  x - x_0 &= \frac{1}{2}(v - v_0)t
\end{align*}
\]
acceleration due to gravity: $g = 9.80 \text{ m/s}^2$
for uniform circular motion: \[|\vec{a}_c| = a_c = \frac{v^2}{r}\]
\[
\begin{align*}
  \sum \vec{F} &= m\vec{a} \\
  f_k &= \mu_n n \\
  f_s &\leq \mu_n n \\
  \hat{i} \cdot \hat{j} &= \hat{j} \cdot \hat{k} = \hat{k} \cdot \hat{i} = 1 \\
  \hat{i} \cdot \hat{j} &= \hat{j} \cdot \hat{k} = \hat{k} \cdot \hat{i} = 0 \\
  \hat{i} \times \hat{j} &= \hat{k} \\
  \hat{j} \times \hat{k} &= \hat{i} \\
  \hat{k} \times \hat{i} &= \hat{j} \\
  \hat{i} \times \hat{i} &= \hat{j} \\
  \hat{j} \times \hat{j} &= \hat{k} \\
  \hat{k} \times \hat{k} &= \hat{i} \\
  \hat{A} \cdot \hat{B} &= AB \cos \theta \\
  \left| \hat{A} \times \hat{B} \right| &= AB \sin \theta \\
  W &= \vec{F} \cdot \Delta \vec{r} \\
  W &= \int \vec{F} \cdot d\vec{x} \\
  P_{\text{proj}} &= W / \Delta t \\
  P_{\text{avg}} &= \Delta E / \Delta t \\
  \vec{p} &= \vec{F} \cdot \vec{v} \\
  K &= \frac{1}{2} m \vec{v}^2 \\
  U_g &= mgL \\
  U_s &= \frac{1}{2} kx^2 \\
  C &= 2\pi R
\end{align*}
\]
Roots of \[ax^2 + bx + c = 0\] are:
\[x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}\]
\[\frac{d(At^n)}{dt} = nA t^{n-1}\]

For constant $\alpha$
\[
\begin{align*}
  \omega &= \omega_0 + \alpha t \\
  \theta &= \theta_0 + \alpha t + \frac{1}{2} \alpha^2 t^2 \\
  \omega^2 &= \omega_0^2 + 2\alpha(\theta - \theta_0)
\end{align*}
\]
\[
\begin{align*}
  F_{\text{spring}} &= -kx \\
  W_{\text{by spring}} &= -\frac{1}{2} kx_0^2 - \frac{1}{2} kx^2 \\
  E_{\text{mech}} &= K + U \\
  \Delta K &= K_f - K_i = \sum W_{\text{other forces}} \cdot f \cdot \Delta t \\
  W_{\text{by cons. force}} &= -\Delta U = -(U_f - U_i) \\
  -f \cdot \Delta t &= \Delta K + \Delta U = \Delta E_{\text{mech}} \\
  \Delta E_{\text{int}} &= f \cdot \Delta t \\
  F_x &= -\frac{dU}{dx} \\
  \dot{p} &= m\vec{v} \\
  \vec{I} &= \Delta \vec{p} \\
  \sum F_{\text{external}} &= \frac{d\vec{p}_{\text{total}}}{dt} \\
  \vec{R}_{\text{of mass}} &= \sum m_i \vec{r}_i = \sum m \vec{r}_1 + m \vec{r}_2 + \cdots + m \vec{r}_N \\
  \vec{I} &= \sum m_i \vec{r}_i^2 = m \vec{r}_1^2 + m \vec{r}_2^2 + \cdots + m \vec{r}_N^2 \\
  v &= ra \omega \\
  a &= ra \alpha \\
  K &= \frac{1}{2} I \omega^2 \\
  \tau &= \vec{r} \times \vec{F} = r F \sin \theta \\
  L &= \vec{r} \times \vec{p} = r p \sin \theta \\
  I &= I_{\text{of mass}} + MD^2 \\
  L &= I \omega \\
  L &= mvd \\
  \sum \tau &= Ia \\
  f &= 1/T = \omega/(2\pi) \\
  T &= 2\pi \sqrt{m/k} \\
  T &= 2\pi \sqrt{L/g} \\
  x(t) &= A \cos(\omega t + \phi) \\
  v(t) &= -\omega A \sin(\omega t + \phi) \\
  a(t) &= -\omega^2 A \cos(\omega t + \phi)