

# Introduction to Simple Harmonic Motion



# Introduction

## Hooke's Law

For a mass on a spring system, by **Hooke's Law** we know that there will be a restoring force acting on the mass when it is displaced from its equilibrium position. This restoring force is written as  $F_x = -kx$ , since the force is proportional to the displacement  $x$ , and *opposite* in direction. In Hooke's Law,  $k$  is the spring constant.

For an oscillating mass  $m$  on a spring system, **angular frequency**  $\omega$  is related to the mass and spring constant with the relationship

$$\omega = \sqrt{\frac{k}{m}}$$

# Introduction

## Simple Harmonic Motion

In this experiment you will be investigating some of the basic properties of periodic motion. In general, any motion that repeats itself at regular intervals is called oscillatory, periodic, or harmonic motion. Examples of periodic motion can be found almost anywhere; boats bobbing on the ocean, the pendula of grandfather clocks, and vibrating violin strings.

**Simple Harmonic Motion** (SHM) satisfies the following properties:

- Motion is about an equilibrium position.
- Motion is periodic.

The main purpose of this laboratory is to become familiar with some of the parameters of oscillatory motion, and to investigate the dynamic relationships that define SHM. You will also investigate Hooke's Law.

# Introduction

For a system undergoing SHM, the displacement with respect to the equilibrium position,  $x$  (as a function of time,  $t$ ), can be described by

$$x(t) = A \cos(\omega t)$$

where  $A$  is the maximum displacement or **amplitude** of the motion and  $\omega$  is the **angular frequency** of the motion. The period is the time taken for one complete oscillation, and can be expressed mathematically as

$$T = \frac{1}{f}$$

where  $f$  is the **frequency** of the motion.

The frequency is defined to be the number of oscillations that the system completes in one second. The symbol for frequency is  $f$  and its SI unit of measure is hertz ( $Hz$ ),  $1 Hz = 1/s$  (1 oscillation per second).

# Introduction

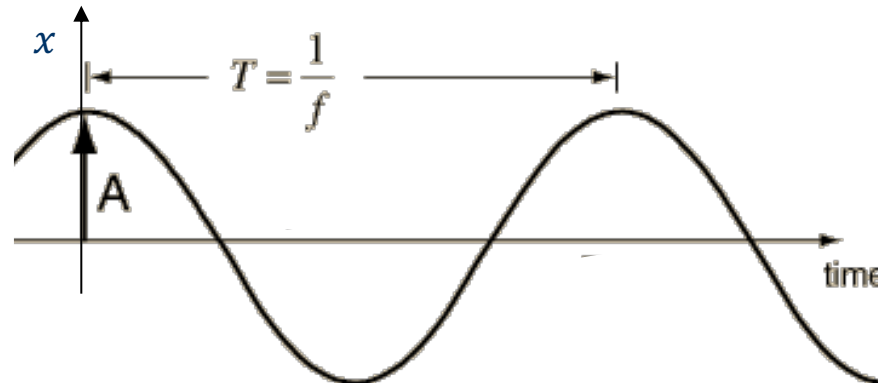
The **angular frequency**  $\omega$  is defined in terms of the frequency  $f$

$$\omega = 2\pi f$$

We may combine the above equations to find the equation for period to be

$$T = 2\pi \sqrt{\frac{m}{k}}$$

A sketch of  $x$  vs  $t$  looks like



# Objectives

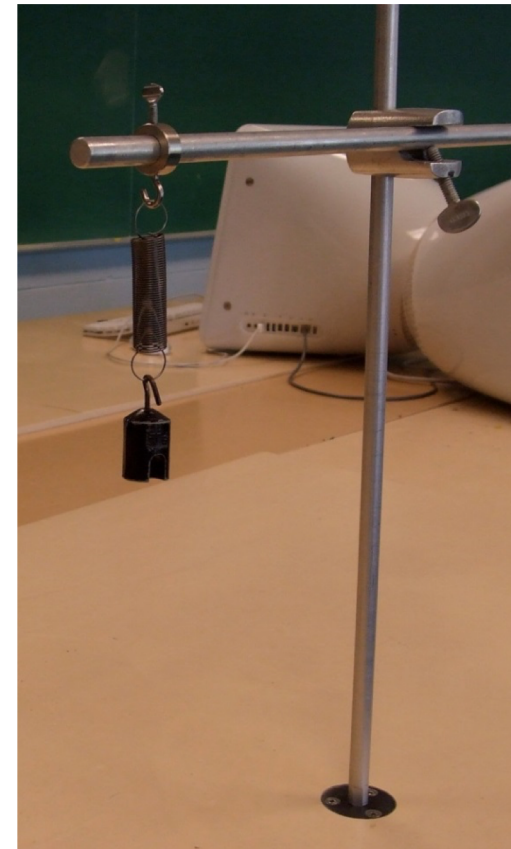


- In this experiment you will determine the force constant of a spring.
- You will measure the period of simple harmonic motion for six different masses and graph the results.
- You will use a motion detector to generate graphs of position, velocity, and acceleration for simple harmonic motion.

# Apparatus and Setup

The apparatus for this experiment consists of:

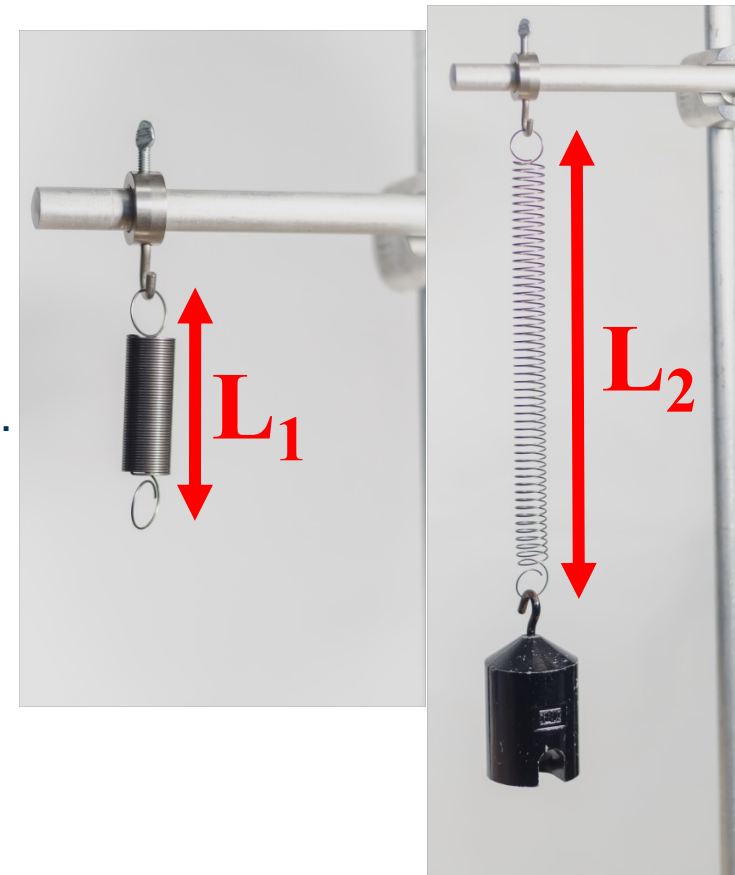
- hook
- lab pro
- masses
- metre stick
- motion sensor
- rods and clamps
- spring
- stopwatch



The setup is shown in the right picture.

# Finding the Spring Constant

- Using the metre stick, measure the length,  $L_1$ , of the spring (from the top hook to the bottom hook as shown).
- Record this value and its uncertainty in **Table 1**.
- Hang the 500.0  $g$  mass from the spring.
- Calculate the weight, in newtons, of the 500.0  $g$  mass and record in **Table 1**.
- Measure the new length,  $L_2$ , of the spring.
- Record this value and its uncertainty in **Table 1**.
- **NOTE:** Make both length measurements from the top of the top loop to the bottom of the bottom loop as shown.



LW

LW

LW



# Finding the Spring Constant

From Newton's second law, the magnitude of the spring force (Hooke's Law states that  $(\vec{F}_x = -k\Delta\vec{x})$ ) will equal the weight hanging from the spring, i.e.

$$W = k\Delta x = k(L_2 - L_1)$$

where  $W$  is the weight of the 500.0 g mass and  $L_2 - L_1$  is the change in length due to the weight of the 500.0 g mass.

**QUESTION 1:** Use your results from Table 1 to determine the spring constant. Don't forget units and uncertainty. You may take  $\delta m = 0.0002 \text{ kg}$  and  $\delta g = 0$ .

# Period squared vs mass

- Hang a  $200.0\text{ g}$  mass from the spring.
- Lift the mass a few cm upwards and gently release to set the mass oscillating.
- Use the stopwatch to time 10 oscillations and record this time in **Table 2**.
- Have your partner repeat the timing of 10 oscillations and record the result in **Table 2**.
- Increase the hanging mass by  $50.0\text{ g}$  and have both partners time 10 oscillations, recording the results in **Table 2**.
- Continue the procedure, increasing the mass by  $50.0\text{ g}$  each time until  $450.0\text{ g}$ , recording your results in **Table 2**.

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# Period squared vs mass

LW

- For each mass, determine the average time for 10 oscillations and record the results in **Table 2**.

LW

- Using the average time, determine the period of oscillation and record your results in **Table 2**.

LW

- Calculate period squared and record your results in **Table 2**.

# Period squared vs mass



For help with these questions, refer to the Plotting and Interpreting Graphs section of your Laboratory Workbook.



**QUESTION 2:** Write the equation for the slope of a period squared ( $T^2$ ) vs mass ( $m$ ) graph. What do you predict for the intercept? Explain.



Remember that the equation of a straight line is  $y = mx + b$ .

Compare this to the physics equation which relates mass to period squared to find the physical meaning of your slope and the intercept.



**QUESTION 3:** Using your result from Question 2, sketch the predicted shape of a period squared vs mass graph.



Do not use data! This is a theoretical prediction.

# Period squared vs mass

Click the icon below to launch Graphical Analysis



Use Graphical Analysis to plot Period Squared ( $y$  – axis) vs Mass ( $x$  – axis).

Create a linear fit to your graph by clicking **Analyze** then **Linear Fit**. Find the uncertainty in your slope and intercept by double clicking on the pop up box and clicking the option to “Show uncertainty”.

**LW**

Record the slope, intercept, and uncertainties in **Table 3**.

**P**

Print your graph by a) Click **File** then **Page Setup**, select **landscape** orientation and click **OK**; b) Click **File** then **Print**, change number of copies to **2** (one for each partner) and **Pages** from: 1 to 1, clicking **Print** then **Print**.

## Period squared vs mass

Q

**QUESTION 4:** Use your slope expression from Question 2 to determine the spring constant  $k$ . Be sure to include units and uncertainty.

Q

**QUESTION 5:** Write the range of values of  $k$  and determine if it agrees with your results from Question 1? Comment on their agreement.

*e.g.* Writing the range of  $L = 1.0 \pm 0.1 \text{ cm}$  means the value ranges from  $0.9 \leq L \leq 1.1$ . If values agree, their ranges should overlap.

Q

**QUESTION 6:** How does your intercept value compare with your predicted value? Comment on their agreement.

# $x(t)$ , $v(t)$ , and $a(t)$ for SHM

Click on the icon to launch *LoggerPro*.



*LoggerPro* should display 3 graphs:

*position vs time*

*velocity vs time*

*acceleration vs. time*


If *LoggerPro* does not contain these graphs, consult an instructor.



# $x(t)$ , $v(t)$ , and $a(t)$ for SHM

- Hang the aluminium mass from the spring as shown in the photo.
- Place the motion sensor directly under the mass.
- Plug the motion sensor into DIG/SONIC 1 port of the LabPro.

Careful with the placement! The motion sensor must be directly under the mass in order to work properly.

- With the mass still, click **Experiment**, then **Zero**, to zero the motion sensor.
- Lift the mass a few centimeters upward and gently release to set the mass oscillating.
- Allow oscillations to stabilize, click  in *LoggerPro* to collect data.





# $x(t)$ , $v(t)$ , and $a(t)$ for SHM

Examine your data. Does it have the expected form? If your data is not satisfactory, repeat until it is.

Have an instructor check your graph and initial your book.

Print your graphs by a) Click **File** then **Page Setup**, select **landscape** orientation and click **OK**; b) Click **File** then **Print**, change number of copies to **2** (one for each partner) and **Pages** from: 1 to 1, clicking **Print** then **Print**. Attach **both** printed pages into Laboratory Workbook.



# Data analysis - $x(t)$ , $v(t)$ , and $a(t)$

LW

- On your printout, draw a straight **vertical** line through the first maximum on  $x$  vs  $t$ , extended through all plots vertically. Label this line “1”.

LW

- Line number “2” will go through the zero position to the right of the first maximum of the position graph.

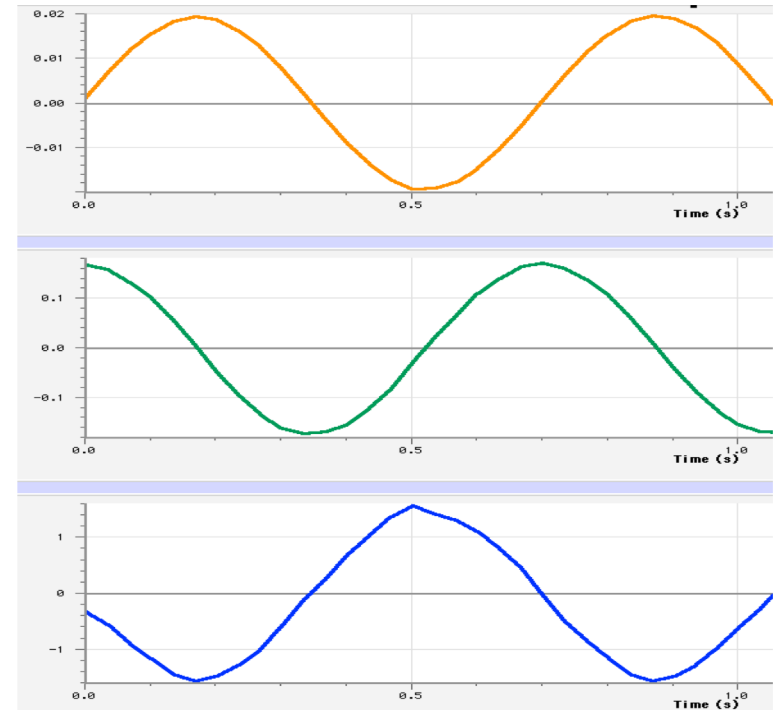
LW

- Line number “3” will go through the minimum of the position graph.

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- Line number “4” will go through the next zero on the position graph.

1 2 3 4



# Data analysis - $x(t)$ , $v(t)$ , and $a(t)$

Q

**QUESTION 7:** From your graph printout, indicate which numbered vertical line corresponds to the following conditions:

- A) Displaced upwards and stopped.
- B) Travelling upward at maximum speed.
- C) Zero acceleration and moving up.
- D) At equilibrium point and travelling down.
- E) Maximum acceleration and displaced down.

# Summary

**QUESTION 8:** Give at least two sources of error in this experiment and classify them as random or systematic.

- Check that you have completed all the tables of your Laboratory Worksheet.
- Make sure that you have answered all the questions completely.
- Attached to your Laboratory Worksheet should be the following graphs:
- Period Squared vs Mass (Graphical Analysis)
- Position, Velocity, and Acceleration vs Time (LoggerPro)