Equilibrium of a Rigid Body

Introduction

A rigid body is in equilibrium when it is not undergoing a change in rotational or translational motion. This equilibrium requires that two conditions must be met.

The **first condition** is related to the translational motion. **The vector sum of the forces on the body must be zero:**

$$\sum \vec{F} = 0$$

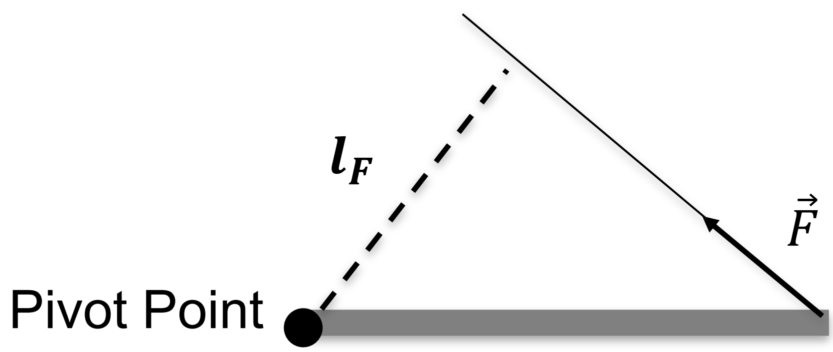
The second condition is related to the rotational motion. When the forces do not act through a common point or pivot, they may cause the body to rotate, even though the vector sum of the forces may be zero. This requires introducing the idea of torque due to a force. A net torque will cause a body, initially at rest, to undergo rotation.

The **second condition** for static equilibrium is: **The sum of the all the torques (due to each of the forces on the body) must be zero**:

$$\Sigma \vec{\tau} = 0$$

Introduction

The torque due to a force depends on and is proportional to both the magnitude of the force and the lever arm. The lever arm $\boldsymbol{l_F}$ depicted in the diagram below, is the perpendicular distance between the line of force (force vector) and the pivot point as illustrated in the diagram below.



The **magnitude** of the torque τ, due to a single force F, is defined as the force F multiplied by the lever arm l_F,

$$\boldsymbol{\tau = F l_F}$$

Introduction

Direction of torque: Torque due to a force that acts to rotate a body in the counter-clockwise direction is said to be positive, while clockwise torques are negative.

The pivot point may be chosen to **be any point** on the rigid body in equilibrium.

Objectives

In this laboratory you will investigate the concepts of torque and equilibrium.

Your objectives are:

1. Become familiar with the concept of lever arm and torque.
2. Become familiar with the two conditions for static equilibrium.

You will also determine the weight and center of gravity of a meter stick as well as learn how to balance forces to maintain static equilibrium.

You will investigate the equilibrium conditions for a case where some of the forces are at an angle to the rigid body.

Apparatus

You should have this apparatus at your station:

- Two meter sticks
- Pivot stand
- Pivot clamp
- Force probe
- Rods
- Set square
- Mass set
- Strings

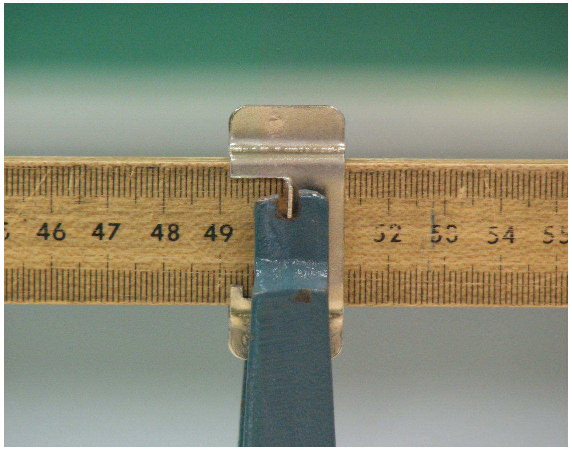
If anything is missing please let us know.

Definition of Center of Gravity

The center of gravity is the single point through which the weight of the rigid body acts. If we apply a force directly opposing the force of gravity at this point, we can balance the object.

Finding the Center of Gravity

- Slide the metal clamp on to the meter stick near the middle.
- Place the meter stick on the pivot stand using the pivot clamp located near the center.
- Adjust the position of the pivot clamp on the meter stick until the meter stick is balanced and level. Tighten the set screw on the clamp to lock the position.
- If the stick oscillates slowly by equal amounts on each side of the pivot, then it is balanced.
- Do not move the clamp for the rest of the lab.



Center of Gravity and Weight

LW
- Take the meter stick off the stand and measure the position of the pivot clamp on the meter stick. IMPORTANT: Read the positon of the pivot clamp through its center. Estimate your uncertainty.

LW
- Record the position of the center of gravity and its uncertainty in **Table 1**.

LW
- Measure the mass of the meter stick, with clamp still attached, on a triple beam balance. Record this value and its uncertainty in **Table 1**.

LW
- Calculate the weight of the meter stick along with its experimental uncertainty. Record these values in **Table 1**.
- Note: You may assume $\delta g = 0$.

Balancing the forces

Using a loop of string, hang a **20g mass at the 70cm** mark on the meter stick.

Q

QUESTION 1: Approximately, where would you expect to hang a 10g mass to keep the meter stick balanced? Explain your reasoning.

Adjust the position of the 10g mass on the meter stick to balance it and 20g mass on the pivot stand. Both masses should be suspended from the table when balanced.

LW

Record the distance from the pivot stand to the 10g mass as l_{10} in Table 2.

LW

Record the distance from the pivot stand to the 20g mass as l_{20} in Table 2.

Q

QUESTION 2: Did this result match your expectation from earlier? Explain.

Balancing the forces

Q

QUESTION 3: Sketch a free body diagram showing all the forces acting on the meter stick. Include the forces due to the 10g and 20g masses. Note that the forces need to go in their correct positions (i.e. the points at which the forces act).

Recall the expression for torque is given as:

$$\tau = Fl$$

Where τ is the torque, F the force, and l the lever arm. We will use this definition to find the torque due to each of the masses.

Balancing the forces

Q

QUESTION 4: Using the clamp position as the pivot point, determine the torque and its uncertainty due to:

a) the $10\ g$ mass.

b) the $20\ g$ mass.

NOTE: You can take δm to be $0.1\ g$.

Q

QUESTION 5: Are the magnitudes of the torques equal within uncertainty? To answer this, calculate the ranges of the two torques and see if the ranges overlap.

Q

QUESTION 6: Given your answers to Questions 4 and 5, is the second condition for equilibrium satisfied ($\Sigma\tau = 0$)? Comment. Note that clockwise torques are negative.

Q

QUESTION 7: What is the weight in newtons of the $200\ g$ mass?

Equipment Setup

The force probe will not display the correct force in newtons unless we calibrate it by applying two forces of known values.

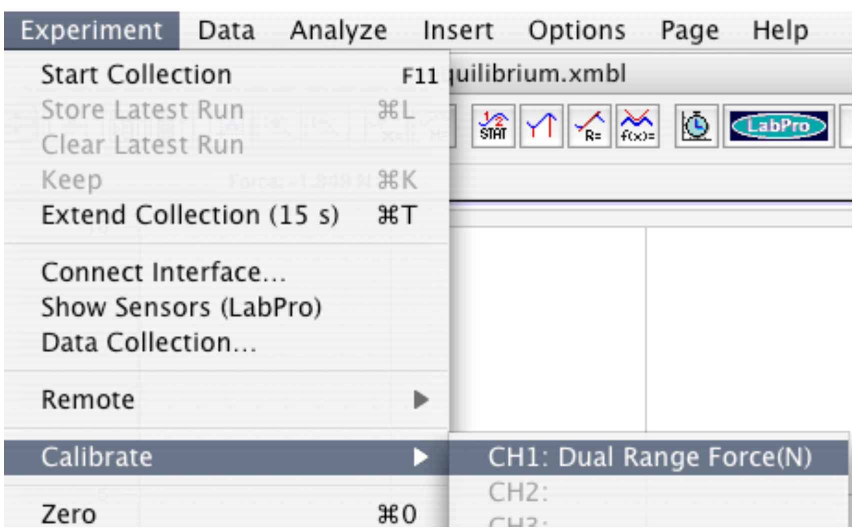
- Clamp the short aluminum rod to the long rod screwed into the bench.
- Mount the force probe onto the short rod as shown and tighten the thumbscrew. The hook should be pointing straight down.
- Ensure the force probe is set to $5\ N$ or $10\ N$ and not to $50\ N$. Plug the Force Probe into **CH 1** on the LabPro.
- Launch *Logger Pro* by clicking on the icon below.

CLICK HERE

CLICK HERE

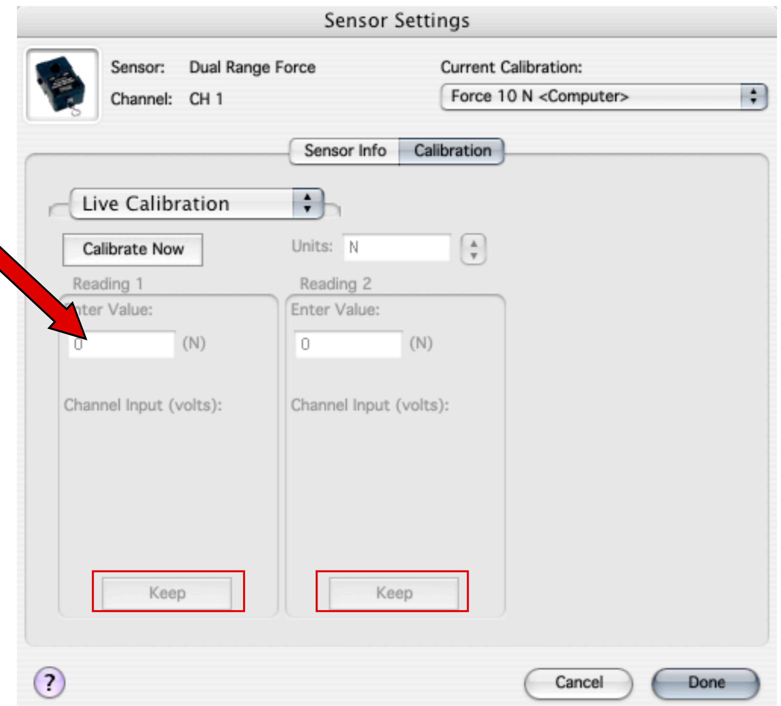
Force Probe Calibration

- The *Logger Pro* window should contain a graph of Force vs Time. If it does not, consult an instructor.
- The force probe needs to be calibrated before you can use it for measurements. A 200 *g* mass is to be used for calibration.



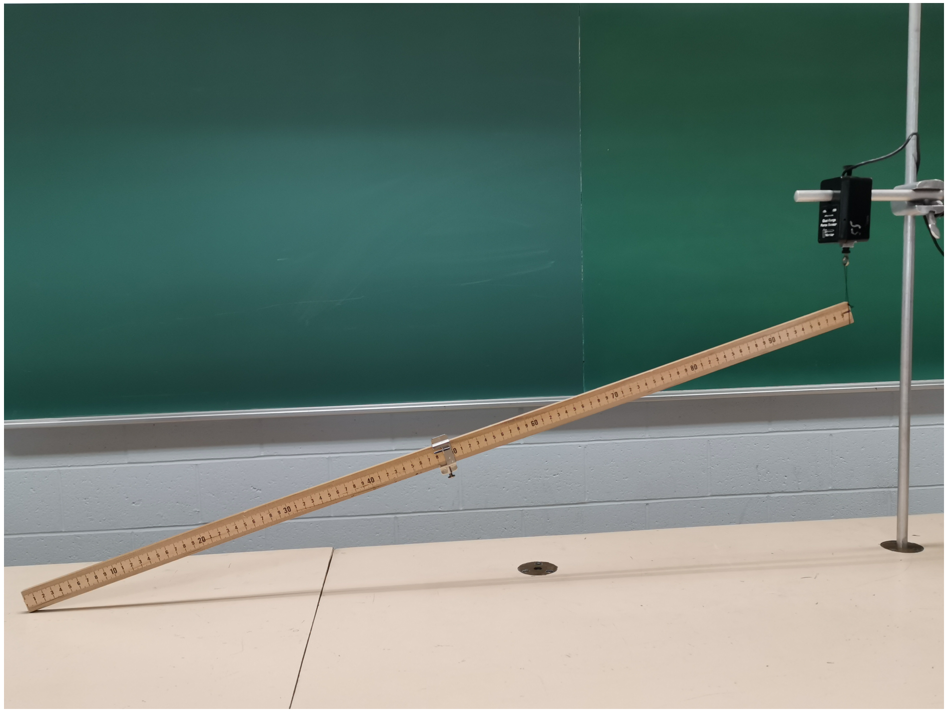
Calibrating the Force Probe

- Click **Calibrate Now**. Do two readings:
  1. With nothing hanging on the force probe, enter **0** in the Value box, then click **Keep**.
  2. Hang the $200\ g$ mass and enter its weight (in newtons) in the Value box, then click **Keep**.
- Channel Inputs should be different.
- Click **Done**.
- Check your calibration: press **Collect** with $200\ g$ on the probe. If the weight is not close to $1.96\ N$, recalibrate!
- You are now ready to collect data.



Angled Meter Stick

- Remove the metal stand.
- Set up the meter stick as shown. **Make sure that the force probe and string are vertical.**



Free Body Diagram

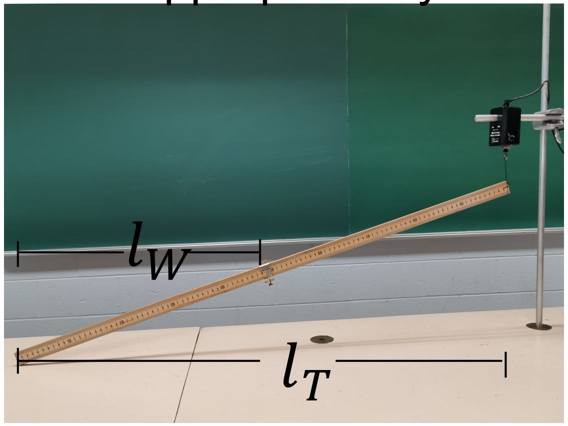
Q

QUESTION 8: How do you know the meter stick is in static equilibrium?

Q

QUESTION 9: Draw a sketch of the meter stick and indicate the three forces (F_N, W and T) acting on it as well as their lever arms.

- Choose the pivot point as the contact point between the meter stick and the bench top, and
- Indicate each of the forces
- Label them with appropriate symbols (for example W, l_T, etc.).



Lever Arm

QUESTION 10: a) Write the equation for the torque due to the weight.

b) Write the equation for the torque due to the tension.

Include the correct sign based on the convention for torque.

- You will now measure the lever arms for the weight l_W and the tension l_T.
- Use a set square to find the point on the lab bench directly below the point at which the tension acts. Mark this location as precisely as possible using a pencil mark on masking tape (do NOT mark on the bench). Then measure l_T.
- Similarly, measure l_W.

QUESTION 11: Record the lever arms l_T and l_W with their experimental uncertainties.

Measuring the Tension

LW

- In Logger Pro, press **Collect**. It will start collecting multiple force probe measurements for a few seconds and then stop.
- Click the statistics button, ½ This will give the average tension T_{avg}, standard deviation, and the number count or number of measurements, N. Place this information in **Table 3**.

Q

QUESTION 12: Calculate the standard error in T_{avg} from **Table 3.** The standard error is the uncertainty in the average. Record T_{avg} and its uncertainty in the form $T_{avg} \pm \delta T_{avg}$.

CP

Have an instructor check your value for the tension and its experimental uncertainty.

Angled Meter Stick - Analysis

Q **QUESTION 13:** Why is the torque due to the force F_N exerted by the table zero?

Q **QUESTION 14:** Refer to Questions 10, 11 & 12, calculate the torque due to the tension force T. Include the experimental uncertainty in the torque.

Q **QUESTION 15:** Refer to Questions 10, 11 & Table 1, calculate the torque due to the weight of the meter stick. Include the experimental uncertainty in the torque.

Q **QUESTION 16:** Determine the sum of the torques and find the experimental uncertainty. Is this sum equal to zero within experimental uncertainty?

Summary and Conclusion

You will now reflect on what this lab demonstrated about static equilibrium.

Q

QUESTION 17: Using what you have learned in this lab about static equilibrium, calculate the force exerted by the table F_N in Q9. (from the angled meter stick section of the lab).

Q

QUESTION 18: Identify two sources of uncertainty in this experiment. Are these random or systematic?

- **Close all applications and log out.**
- **Put away all lab equipment and return your chairs/lab stools.**
- **Submit your laboratory workbook.**