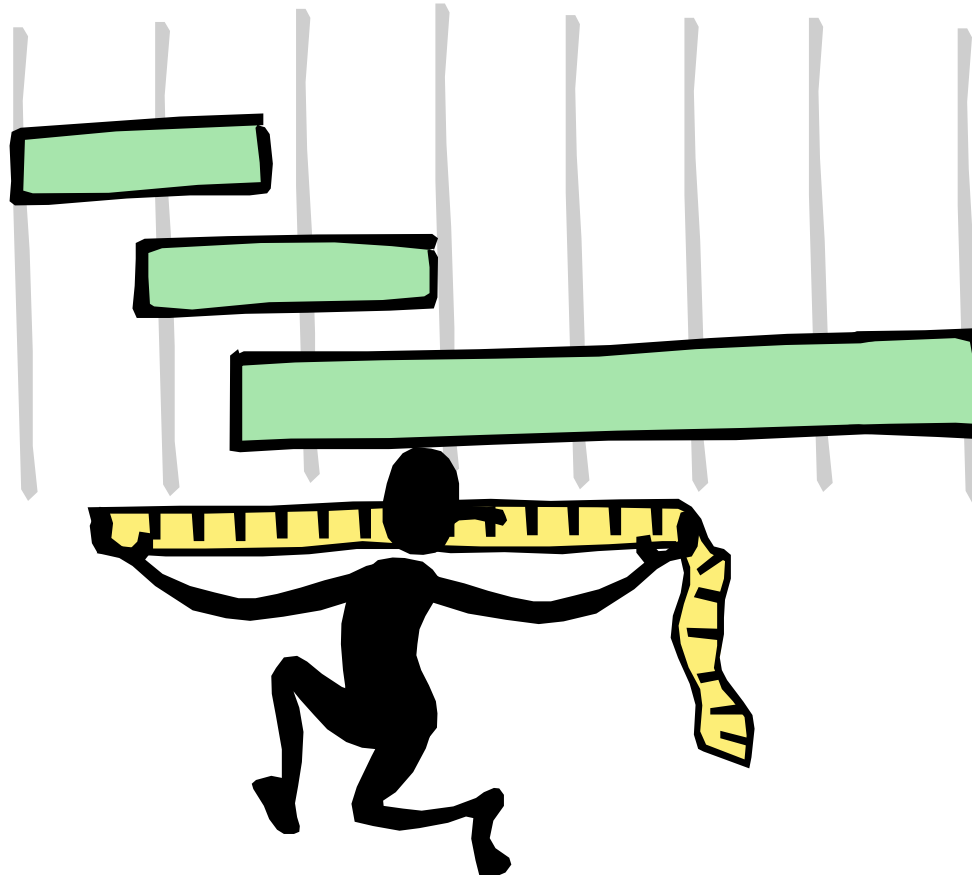


# Measurement and Uncertainty



## **Introduction**

Any experimental measurement or result has an uncertainty associated with it. In today's lab you will perform a set of very simple measurements. You will have to estimate the random uncertainty associated with each of them. As a rule of thumb the precision of your measuring device (for example a ruler) is always a very good starting value for your uncertainty.

Furthermore you will be asked to perform some calculations using the values you just measured. The results of those calculations will also have an uncertainty associated with them. To obtain those values you will have to follow a set of rules. They are explained on this book in "Making Measurements in Physics" section.

Finally you will collect a set of 10 measurements of the same quantity from your classmates and asked to calculate their average. That result also has an uncertainty associated with it. Your instruction will tell you how to calculate it.

**Introduction**

Also recall that the perimeter  $P$  of a rectangular object is the total length of all the sides, i.e.

$$P = L + L + W + W = 2L + 2W$$

Similarly the area  $A$  of a solid rectangular object is

$$A = L \times W$$

where  $L$  is the length and  $W$  is the width of the object.

**Introduction**

Below is a summary of some of the rules you will need for your uncertainty calculations:

***Addition/Subtraction Rule***

When adding or subtracting values with uncertainties, add the absolute uncertainty of each value to find the result absolute uncertainty.

IF  $z = x + y$  OR  $z = x - y$ , THEN

$$\delta z = \delta x + \delta y$$

**Example:** Given  $A = (122 \pm 5) m$ ,  $B = (207 \pm 2) m$ , find  $C = A + B$

$$C = A + B = 122 m + 207 m = 329 m$$

$$\delta C = \delta A + \delta B = 5 m + 2 m = 7 m$$

$$C = C \pm \delta C = 329 m \pm 7 m = (329 \pm 7)m$$

## Introduction

### *Multiplication/Division Rule*

When multiplying or dividing values with uncertainties, add the relative uncertainty of each value to find the result relative uncertainty.

IF  $z = x \times y$  OR  $z = \frac{x}{y}$ , THEN

$$\frac{\delta z}{z} = \frac{\delta x}{x} + \frac{\delta y}{y}$$

**Example:** Given  $A = (122 \pm 5) m$ ,  $B = (207 \pm 2) m$ , find  $C = A \times B$

$$C = A \times B = 122 m \times 207 m = 25254 m^2$$

$$\frac{\delta C}{C} = \frac{\delta A}{A} + \frac{\delta B}{B} = \frac{5}{122} + \frac{2}{207} = 0.0506$$

$$\delta C = \left(\frac{\delta C}{C}\right) \times C = 0.0506 \times 25254 m^2 = 1279 m^2$$

$$C = C \pm \delta C = 25254 m^2 \pm 1279 m^2 \cong (2500 \pm 1000) m^2 \text{ or } (2.5 \pm 0.1) \times 10^4 m^2$$

**Introduction**

**Note:** The uncertainty  $\delta C = 1279 \text{ m}^2$  has been rounded to one significant figure, so it becomes  $1000 \text{ m}^2$ .

**Important:** The only difference in the two rules is one uses absolute experimental uncertainty and one uses relative experimental uncertainty. If you use the multiplication rule, you can always get the absolute experimental uncertainty from the relative experimental uncertainty.

## Objectives



The objectives of this lab are to:

- Learn how to **perform simple measurements**.
- Learn how to **find experimental uncertainties** in your measurements.
- Learn how to correctly **write down your measurements** with their uncertainties.
- Learn how to obtain the **uncertainty in a calculated value** (for example a perimeter, an area or average).
- Learn the difference between **precision** and **accuracy**.

**Experimental Uncertainty in a Single Measurement**Q

**QUESTION 1:** What is your estimate of the best possible precision of the ruler?

!

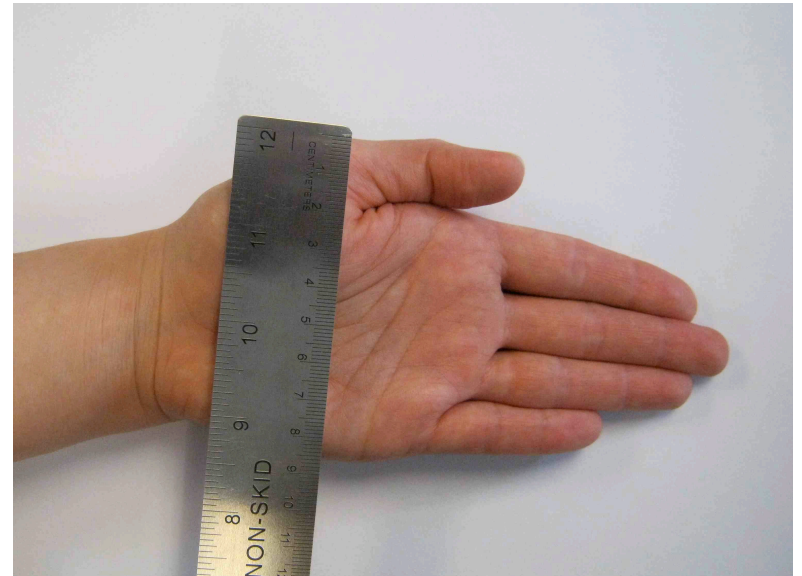
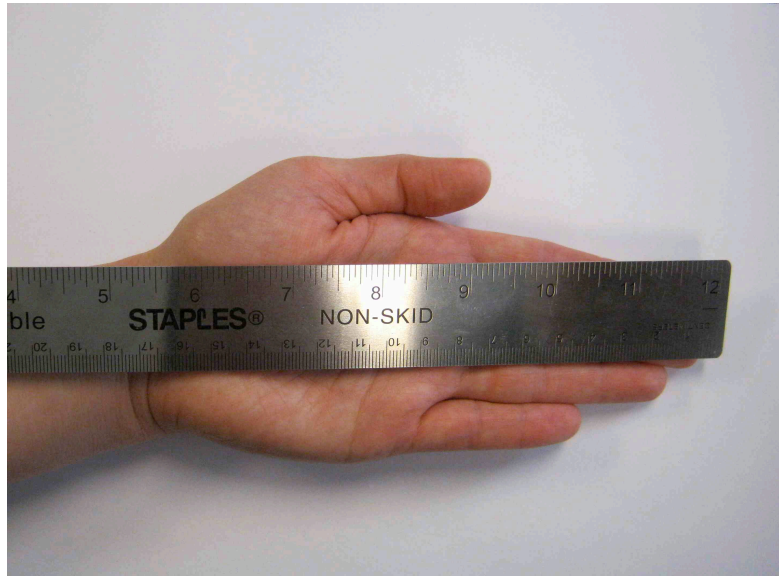
Hint: For a device with a scale its precision can be determined by figuring out what is the smallest division on the scale



### Experimental Uncertainty in a Single Measurement

**LW**

Measure the length and the width of the palm of your hand with the ruler provided. Record your results in **Table 1** in your lab book. Make sure that you include units.




## Estimating Experimental Uncertainty


 LW

The measurements you just recorded in **Table 1** have uncertainties associated with them. Their values are largely determined by the precision of the measuring device, i.e. the ruler in this case.


In general, the precision of an instrument is a very good choice for the uncertainty of a single measurement performed with it. However, certain qualities of the object being measured can also influence the uncertainty, usually making it bigger.



Remember that the uncertainty is a reflection of your confidence that the measurement is inside a certain range of values.



Are the edges of your object (i.e. a hand for this experiment) sharp and well defined or are they not clearly defined?



Ask yourself how reliable is your measurement: are you confident to 0.2 mm, 0.5 mm, 1 mm, 2 mm, or more? Whatever you decide will be your uncertainty.

## Estimating Experimental Uncertainty (cont'd.)

Q

**QUESTION 2:** Write down the estimated uncertainties in your length and width measurements based on the considerations discussed above.

Q

**QUESTION 3:** Comment on any difference between the precision of the ruler and your estimated uncertainties in the measurements.

Q

**QUESTION 4:** Write down the length and width of your hand together with your estimated uncertainty in the form  $x \pm \delta x$ . Be sure to include units.

Your uncertainty should have only one significant figure and your measurement value should have the same number of decimal places as your uncertainty.

!

Notice that your length measurement is larger than your width measurement. To determine which of those measurements is more precise you have to calculate their corresponding **relative uncertainties**. The smaller the relative uncertainty the more precise the measurement is.

## Relative Uncertainty

**Q**

**QUESTION 5:** Calculate the relative uncertainties for the length and width of your hand. Which relative uncertainty is larger?

**Q**

**QUESTION 6:** According to your answer to Question 5 which measurement is more precise?

**CP**

**Have your instructor check your measurements and your relative uncertainties and sign your Laboratory Workbook.**

## Combining Quantities and Comparing Results

You will now calculate the perimeter and area of your hand treating it as a rectangle of length  $L$  and width  $W$  (see Table 1). The perimeter is defined as the distance around a two dimensional shape.

**Q**

**QUESTION 7:** Calculate the perimeter  $P$  of your hand. Be sure to include units. Show your workings.

In order for you to find out the absolute uncertainty of the perimeter you will have to use the addition/subtraction rule stated in the Introduction section of this lab.

**Q**

**QUESTION 8:** Calculate the absolute uncertainty in the perimeter of your hand.

## Combining Quantities and Comparing Results

**Q**

**QUESTION 9:** Write down the perimeter of your hand in the form  $x \pm \delta x$  following the rules outlined in the introduction section of this experiment.

**Q**

**QUESTION 10:** Why is the absolute uncertainty in perimeter larger than the absolute uncertainties in length or width?

Treating your hand as a rectangle wasn't the most accurate method of finding out its perimeter. You will now use a piece of string to find a more accurate value of that quantity.

## Measuring Perimeter

You will now measure the perimeter of your hand.

Use a string to make an outline of your hand, keeping it as close as possible to the outside of your hand. Be sure to close the loop across your wrist!

When done, use a meter stick measure the length of string corresponding to the perimeter of your hand.


Record this length of the string in **Table 2** of your Laboratory Workbook.

Estimate the uncertainty of this measurement and record the value in **Table 2**.



## Comparing Results

One of the most important things about experimental uncertainties is how we use them to see if two or more quantities can be considered equal.



If you have two values for a quantity  $x$  say  $x_1 \pm \delta x_1$  and  $x_2 \pm \delta x_2$  then these two numbers **agree within experimental uncertainty** if the range  $x_1 - \delta x_1$  to  $x_1 + \delta x_1$  overlaps with the range  $x_2 - \delta x_2$  to  $x_2 + \delta x_2$ .

**For example:** If two people measured the length of an object and the first person got  $L_1 = (10.6 \pm 0.4) \text{ cm}$  and the other person got  $L_2 = (11.1 \pm 0.2) \text{ cm}$  as their results, do their results agree with each other?

The first person says that the length he measured is in a range:

$$10.2 \text{ cm} \leq L_1 \leq 11.0 \text{ cm}$$

The second person says that the length he measured is in a range:

$$10.9 \text{ cm} \leq L_2 \leq 11.3 \text{ cm}$$

Since the two ranges overlap we can say that the two measurements do agree with each other within experimental uncertainty.



# Comparing Results

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**Q**

**QUESTION 11:** Does the perimeter you measured with the string agree with the perimeter you calculated earlier? Refer back to Question 9. In answering this question, write the range of both quantities as in the example above. Comment on any disagreement between the two.

## Combining Quantities and Comparing Results

You will now calculate the area of your hand and its uncertainty

Q

**QUESTION 12:** Treating your hand as a rectangle, calculate the area  $A$  of your hand. Be sure to include units.

Q

**QUESTION 13:** Using the multiplication/division rule (see Introduction section of this lab) and your answers to Question 4 & 5, calculate the relative uncertainty  $\frac{\delta A}{A}$  for the area  $A$  of your hand.

Q

**QUESTION 14:** Calculate the absolute uncertainty  $\delta A$  for the area  $A$  of your hand and write down your result in the form  $A \pm \delta A$ , following proper rules for significant figures.

## Combining Quantities and Comparing Results

You will now measure the area of your hand with graph paper.

Laying your hand on top of the graph paper in your Laboratory Workbook (keep your fingers together), carefully trace the outline of your hand. Each partner should trace their own hand.

Determine the area of your traced hand by counting the squares ( $1 \text{ cm}^2$  each) inside the outline.

Do your best to estimate the area in the squares which are only partially inside the outline of your hand.

Record this area in **Table 3** of your workbook.

Estimate the uncertainty in the area and record this value in **Table 3**. In estimating this uncertainty, ask yourself how many squares you could be off by.



**Combining Quantities and Comparing Results****Q**

**QUESTION 15:** Briefly explain your method for estimating the uncertainty in the area from the graph paper method.

**Q**

**QUESTION 16:** Does the area you measured with the graph paper agree with the area you calculated in Question 12? In your answer to this question, be sure to write the range of both quantities. Comment on any differences.

## **Experimental uncertainty in multiple measurements**

In this part of the lab, you will gather some data from your lab mates, to find out the average (mean) width and length of 10 first year physics students' hands and its uncertainty.

**LW**

First rewrite your measurements from **Table 1** into the first row of **Table 4** (no need to rewrite their uncertainty).

**LW**

Visit other, nearby students in the lab and ask them for their measurements of length and width from their **Table 1**. Write them down in the remaining rows of **Table 4**.

**LW**

You should record 10 values of length and width in **Table 4**.

## Experimental uncertainty in multiple measurements

Your goal now is to find the average length  $\bar{L}$  and the average width  $\bar{W}$  as well as their absolute uncertainties (i.e. standard errors). You will use a software called Graphical Analysis to do that.



Enter your length values in the table.

Click **Analyze** then **Statistics**.

Record the mean ( $\bar{L}$ ), standard deviation ( $\sigma_L$ ), and point count ( $N$ ) in **Table 4**.

Calculate the standard error and record your result in **Table 4**.

Repeat the steps outlined above for your measurements of the width and record the results in **Table 4**.

**STANDARD ERROR** is the absolute uncertainty of the average value.



## Experimental uncertainty in multiple measurements

Q

**QUESTION 17:** Write the average length and width of the 10 students in the form  $x \pm \delta x$  using results from **Table 4**.

Q

**QUESTION 18:** Does your own hand's length and width (Question 4) agree with the average values in **Table 4**? (Calculate ranges and see if they overlap).

Q

**QUESTION 19:** Identify one source of random uncertainty and one source of systematic uncertainty in your experiment.

!

- Place your lab workbook in your assigned shelf.
- Tidy your workbench.
- Sign out (at the center desk by the door) before you leave.