MEMORIAL UNIVERSITY OF NEWFOUNDLAND
DEPARTMENT OF PHYSICS AND PHYSICAL OCEANOGRAPHY

PHYSICS 2750 FINAL EXAM - WINTER 2007 - April 17, 2007

NAME:____________________________ STUDENT NUMBER:____________________________

INSTRUCTIONS:

1. Put your name and student number on each page.
2. Do ALL 5 questions.
3. Some possibly useful equations and constants are provided on the next page.
4. Use only the paper provided. No other books, notes or papers are permitted.
5. Do not remove examination papers from the examination room.

CONSTANTS AND FORMULAE

$\frac{-\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\psi(x) + U(x)\psi(x) = E\psi(x)$

$\frac{-\hbar^2}{2m}\nabla^2\psi(\mathbf{r}) + U(\mathbf{r})\psi(\mathbf{r}) = E\psi(\mathbf{r})$

$p = h/\lambda$

$m_p = 1.673 \times 10^{-27} kg$

$m_n = 1.675 \times 10^{-27} kg$

$c = 3.00 \times 10^8\ m/s$

$h = 6.626 \times 10^{-34}\ J \cdot s = 4.136 \times 10^{-15}\ eV \cdot s$

$\hbar = 1.055 \times 10^{-34}\ J \cdot s$

$E^2 = p^2c^2 + (mc^2)^2$

$L = \sqrt{l(l+1)}\ \hbar,\ \ L_z = m_l\hbar$

$n = 1, 2, 3, ...;\ \ l = 0, 1, 2, ...(n-1)\ ;\ \ m_l = 0, \pm 1, \pm 2, ... \pm l$

$1\ eV = 1.602 \times 10^{-19}\ J$

$E_n = -13.6(\frac{Z^2}{n^2})\ eV$

$E = p^2/2m$

$m_{^{238}_{92}U} = 238.050738\ u$

$m_{^{234}_{90}Th} = 234.043596\ u$

$m_{^4_2He} = 4.002603\ u$

$m_{^1_1H} = 1.007825\ u$

$1\ u = 1.66 \times 10^{-27}\ kg = 931.494\ MeV/c^2$

$T_{1/2} = 0.693/\lambda$

$\int dx\, e^{ax} = (\frac{1}{a})e^{ax}$

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1. Electron Diffraction. (20 points.)

   (a) Explain how the famous Double Slit Experiment demonstrates the wave-particle duality of electrons. Use sketches.

   (b) If the intensity of quantum particles is given by $I = \Psi\Psi^*$ where $\Psi = |\Psi|e^{i\phi}$ is the particle wavefunction, show that in the case of both slits open:

   $$I_{1+2} = I_1 + I_2 + 2|\Psi_1||\Psi_2|\cos(\phi_1 - \phi_2).$$

   What is the meaning of the last term?

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2. Quantum Mechanics in one dimension. (20 points.)

   A particle is described by the wave function

$$\psi(x) = \begin{cases} \sqrt{\dfrac{2}{a}}\, e^{-x/a} & \text{for} \quad x > 0 \\ 0 & \text{for} \quad x < 0. \end{cases}$$

   (a) Use this wavefunction in the Schrodinger equation for the case $U = 0$ and find the energy of this state.

   (b) Write down and sketch the probability density.

   (c) Show that $\psi(x)$ is normalized.

   (d) What is the probability that the particle will be found between $x = 0$ and $x = a$?

   (e) Write down, but do not evaluate, the expression for $< x >$.

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3. Bohr atom. (20 points.)

   (a) Explain how classical theory leads to the conclusion that the hydrogen atom is unstable.

   (b) Write down three of the four postulates Bohr proposed to explain the hydrogen atom.

   (c) What is the wavelength of the emitted photon when the electronic state of hydrogen changes from $n = 4$ to $n = 2$?

   (d) Calcualte the speed of the recoiling hydrogen atom. Hint: Use the classical law of conservation of momentum.

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4. Quantum Mechanics in three dimensions: Hydrogen-like atoms. (20 points.)

   (a) Explain how separation of variables is useful to solve the Schrodinger equation for the electron in hydrogen.

   (b) Be^{3+} is an one-electron ion with $Z = 4$. What are the energies of the two lowest energy levels ? Enumerate all the electron states (n, l, m_l) corresponding to these two levels. What is the degeneracy of the first excited state ?

   (c) Nitrogen has 7 electrons. Using Hund's rule of maximizing the total spin quantum number S, show the spin state (using up-arrows $\uparrow$ and down-arrows $\downarrow$) of the 7 electrons that go into the 1s, 2s and 2p oribitals.

   (d) Discuss how the quantization of the angle between the angular momentum vector $\mathbf{L}$ and z-axis can be used to illustrate the Correspondence Principle in the limit of large l.

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5. Nuclear Structure. (20 points.)

   (a) Explain in qualitative terms why energies involved in nuclear processes are very much larger (typically by one million times) than energies involved in atomic processes, i.e., explain why nuclear forces need to be so strong.

   (b) Starting with the expression $N = N_o e^{-\lambda t}$, show that the half-life of a radioactive isotope and its decay constant are related by $T_{1/2} = 0.693/\lambda$.

   (c) The alpha decay scheme of ${}^{238}_{92}U$ is given by

$$ {}^{238}_{92}U \rightarrow {}^{234}_{90}Th + {}^{4}_{2}He $$

   (Neglect any radioactivity from the daughter nucleus).

      i. Calculate the energy released (Q) by this decay process.

      ii. Calculate the kinetic energy (in MeV) of the emitted alpha particle. Assume that the ${}^{238}_{92}U$ nucleus is initially at rest. Hint: Use the laws of conservation of energy ($Q = K_{Th} + K_{He}$) and momentum.

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