

THE THIRTY-NINTH W.J. BLUNDON MATHEMATICS CONTEST

Sponsored by
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in cooperation with
The Department of Mathematics and Statistics
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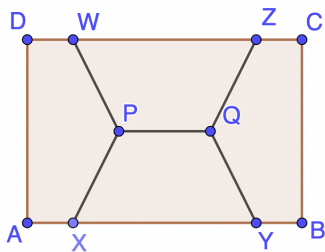
1. If $3x - y = 10$, what is the value of $\frac{8^x}{2^y}$?

2. Show that $10^{25} - 7$ is divisible by 3.

3. Find $x^2 + y^2$ if x and y are positive integers such that

$$xy + x + y = 71 \quad \text{and} \quad x^2y + xy^2 = 880.$$

4. ABCD is a rectangle divided into four parts of equal area by 5 segments as shown in the figure below.



Let PQ and AB be parallel and

$$XY = YB + BC + CZ = ZW = WD + DA + AX$$

If $BC=19$ cm and $PQ=97$ cm, find the length of AB in cm.

5. Given that $\cos(2x) = \frac{5}{13}$ find all possible exact values of $\sin x + \cos x$. Express your answers in the form $a\sqrt{b}/c$, where a, b, c are integers.

6. A turtle and a boy share the same birthday. For six consecutive birthdays the turtle's age is an integral multiple of the boy's age. How old can the turtle be at the sixth of these birthdays, if it can potentially live up to 200 years? List all the possibilities.

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7. a) Find a triple of integers (a, b, c) such that $a + b + c = 0$ and $abc = 2$. Calculate $a^3 + b^3 + c^3$.
- b) Find a triple integers (a, b, c) such that $a + b + c = 0$ and $abc = 6$. Calculate $a^3 + b^3 + c^3$.
- c) Make a conjecture about the way to find $a^3 + b^3 + c^3$ provided the sum $a + b + c = 0$ and the product $abc = K$, where the value of K is given.
- d) Prove your conjecture.
8. I wrote down three positive numbers. Divided by their range, the product of their mode with the difference of their mean and their median was 6:

$$\frac{(\text{Mean} - \text{Median}) \times \text{Mode}}{\text{Range}} = 6.$$

What was the minimum of the three numbers that I wrote?

9. A rectangle ABCD has sides $AB=CD=6$ cm and $AD=BC=8$ cm. Two identical circles are drawn inside of the rectangle such that each circle is tangent to two adjacent sides of the rectangle (AB and BC) and (CD and DA) respectively. In addition, the circles are tangent to each other. Find the exact value (in cm) of the radius of these circles.
10. Alice walks down to the bottom of an escalator that is moving up. She counts 150 steps. Her friend Bob walks up to the top of the escalator and counts 75 steps. Alice's speed of walking (number of steps per unit time) is 3 times Bob's walking speed. How many steps are visible on the escalator at a given time?

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