On 2-factorizations of the complete graph: A mathematical journey through Oberwolfach, Hamilton and Waterloo

Abstract:

In the 1960s, Ringel posed the following problem, known as the Oberwolfach problem. At a conference in Oberwolfach, Germany, attended by $n$ mathematicians, the dining room has round tables of sizes $k_1, k_2, \ldots, k_t$, where $k_1 + k_2 + \cdots + k_t = n$. Is it possible, over the $r$ nights of the conference, for each person to sit next to each other person exactly once? In other words, given a 2-factor $F$ consisting of cycles of lengths $k_1, k_2, \ldots, k_t$, does there exist a 2-factorization of the complete graph $K_n$ in which each 2-factor is isomorphic to $F$?

Several variations of the Oberwolfach problem have since been studied, among the most notable being the Hamilton-Waterloo problem. In this version, the conference has two venues (Hamilton and Waterloo), so we seek to find a 2-factorization of $K_n$ with $\alpha$ factors isomorphic to $F_1$ and $\beta$ isomorphic to $F_2$.

In this talk, we give an overview of these problems, and present some recent results on the Hamilton-Waterloo problem for uniform odd-cycle factors.