Amitsur's conjecture for polynomial H-identities
of H-module Lie algebras

Abstract:

In the 1980's (or even earlier), a conjecture about the asymptotic behaviour of codimensions of ordinary polynomial identities was made by S.A. Amitsur. Amitsur's conjecture was proved in 1999 by A. Giambruno and M.V. Zaicev for associative algebras, in 2002 by M.V. Zaicev for finite dimensional Lie algebras.

Alongside with ordinary polynomial identities of algebras, graded polynomial identities, G- and H-identities are important too. Usually, to find such identities is easier than to find the ordinary ones. Furthermore, the graded polynomial identities, G- and H-identities completely determine the ordinary polynomial identities. Therefore the question arises whether the conjecture holds for graded codimensions, G- and H-codimensions.

We consider a finite dimensional H-module Lie algebra L over a field of characteristic 0 where H is a finite dimensional semisimple Hopf algebra, and prove the analog of Amitsur's conjecture on asymptotic behaviour for codimensions of polynomial H-identities of L. As a consequence, we obtain the analog of Amitsur's conjecture for graded codimensions of any finite dimensional Lie algebra graded by a finite group not necessarily Abelian.

This result is a generalization of the result of the author where he proved the analog of Amitsur's conjecture for G-codimensions for a finite group G and graded codimensions for a finite Abelian group.