Some Exercises in Linear Algebra

1. Each of the following systems of equations has a unique solution. For the first system, there is exactly one value of $x$ and one value of $y$ which satisfy both equations simultaneously. Similarly, there is just one value of $x$, one value of $y$ and one value of $z$ which satisfy the second set of equations simultaneously. Can you find the solutions in each case?

(a) $\begin{align*} x - y &= 5 \\ 3x + 2y &= 10 \end{align*}$  
(b) $\begin{align*} x - y + z &= 6 \\ 2x + y &= 3 \\ x - 3z &= -7 \end{align*}$

2. Neither of the following systems has a solution. Can you prove this?

(a) $\begin{align*} x - y &= 5 \\ 3x - 3y &= 10 \end{align*}$  
(b) $\begin{align*} x - y + z &= 6 \\ 2x - y - 2z &= -2 \\ x - 3z &= -7 \end{align*}$

3. Each of the following systems has infinitely many solutions. In the first system, there are infinitely many values of $x$ and corresponding values of $y$ which satisfy both equations simultaneously. Similarly, there are infinitely many values of $x$, $y$ and $z$ which satisfy the second set of equations simultaneously. Can you find a way to describe the solutions in each case?

(a) $\begin{align*} x - y &= 5 \\ 3x - 3y &= 15 \end{align*}$  
(b) $\begin{align*} x - y + z &= 6 \\ -2x + y + 2z &= 1 \\ x - 2y + 5z &= 19 \end{align*}$