1. Solve the system

\[
\begin{align*}
3^{2x-y} &= 27 \\
2^{3x+2y} &= 32
\end{align*}
\]

2. Prove the identity

\[
\frac{2016^{-x}}{2016^{-x}+1} + \frac{2016^x}{2016^x+1} = 1.
\]

3. (a) If \(\log_3(\log_4(a^3)) = 1\), find \(a\).

(b) Let \(a > 1\). Find all possible solutions for \(x\) such that the following equation holds:

\[
\log_a x + \log_a(x-2a) = 2
\]

4. (a) Prove that

\[
S = 1 + 2 + 3 + \cdots + (n-1) + n = \frac{n(n+1)}{2},
\]

for any natural number \(n \geq 1\). (Hint: Write the sum backwards and forwards and add the results!)

(b) Determine the value of \(n\) for which

\[
2^1 \cdot 2^2 \cdot 2^3 \cdots 2^n = 2^{210}.
\]

5. Determine the number of integer values of \(x\) such that \(\sqrt{2 - (1+x)^2}\) is an integer. Fully justify that you have identified the correct number.

6. Find all values of \(k\) so that \(x^2 + y^2 = k^2\) will intersect the circle with equation

\[
(x - 5)^2 + (y + 12)^2 = 49
\]

at exactly one point.

7. When a two digit number and a three digit number are multiplied, the result is 7777. Find the largest such three–digit number possible.

8. (a) Prove the identity

\[
(sin x)(1 + 2 \cos 2x) = \sin(3x)
\]

You may use the identities

\[
\sin(x + y) = \sin x \cos y + \sin y \cos x \quad \cos(x + y) = \cos x \cos y - \sin x \sin y
\]

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(b) Suppose $n$ is a positive integer. Prove the identity
\[(1 + 2 \cos(2x) + 2 \cos(4x) + \ldots + 2 \cos(2nx))(\sin x) = \sin((2n + 1)x)\]

9. Calculate the value of the product
\[P = \left(1 + \frac{1}{2}\right) \cdot \left(1 - \frac{1}{2}\right) \cdot \left(1 + \frac{1}{3}\right) \cdot \left(1 - \frac{1}{3}\right) \ldots \cdot \left(1 + \frac{1}{n}\right) \left(1 - \frac{1}{n}\right)\]
where $n \geq 1$ is a positive integer.

10. Define the function $f(x)$ to be the the largest integer less than or equal to $x$ for any real $x$. For example $f(1) = 1$, $f(3/2) = 1$, $f(7/2) = 3$, and $f(7/3) = 2$. Let
\[g(x) = f(x) + f(x/2) + f(x/3) + \ldots + f(x/(x-1)) + f(x/x)\]
(a) Calculate $g(4) - g(3)$ and $g(7) - g(6)$.
(b) What is $g(116) - g(115)$?