1. \(2x^2 + 5x - 3 = (2x - 1)(x + 3)\). So \(x = \frac{1}{2}\) and \(x = -3\) are zeros.

\[
P\left(\frac{1}{2}\right) = 0 \quad \Rightarrow \quad \frac{1}{2} + 1 - 10 + \frac{1}{2}a + b = 0 \quad \Rightarrow \quad \frac{1}{2}a + b = \frac{17}{2} \quad \Rightarrow \quad a = 23
\]

\[
P(-3) = 0 \quad \Rightarrow \quad 648 - 216 - 360 - 3a + b = 0 \quad \Rightarrow \quad -3a + b = -72 \quad \Rightarrow \quad b = -3
\]

\[
P(x) = 8x^4 + 8x^3 - 40x^2 + 23x - 3
\]

Then using either synthetic division twice or long division gives

\[
P(x) = (2x - 1)(x + 3)(8x^2 - 12x + 2) = 2(2x - 1)(x + 3)(4x^2 - 6x + 1)
\]

The other two zeros of \(P(x)\) are

\[
x = \frac{6 \pm \sqrt{36 - 16}}{8} = \frac{6 \pm \sqrt{20}}{8} = 3 \pm \frac{\sqrt{5}}{4}
\]

So the product of the zeros of \(P(x)\) is

\[
\left(\frac{1}{2}\right)(-3)\left(\frac{3 + \sqrt{5}}{4}\right)\left(\frac{3 - \sqrt{5}}{4}\right) = \left(\frac{1}{2}\right)(-3)\left(\frac{1}{4}\right) = -\frac{3}{8}
\]

Or, using the known result that the product of the zeros of \(P(x) = a_n x^n + \cdots + a_0\) is \(\frac{a_0}{a_n}\) we get directly, after finding \(b = -3\), that the product of the zeros is \(-\frac{3}{8}\).

2. Since \(\angle CAD = 30\) deg, \(AD = \frac{\sqrt{3}}{2}r\) and \(DC = \frac{1}{2}r\). So the area of the triangle, which is 6 times the area of triangle \(ADC\) is

\[
\text{Area} = 6 \cdot \frac{1}{2} \cdot \frac{\sqrt{3}}{2} r \cdot \frac{1}{2} r = \frac{3\sqrt{3}}{4} r^2
\]

Or, since \(\angle ACB = 120\) deg and the area of the triangle is 3 times the area of triangle \(ACB\) we get

\[
\text{Area} = 3 \cdot \frac{1}{2} AC \cdot CB \sin 120\deg = \frac{3}{2} r \cdot r \cdot \frac{\sqrt{3}}{2} = \frac{3\sqrt{3}}{4} r^2
\]

3. \(2xy - 4x^2 + 12x - 5y = 5\)
\(2xy - 5y = 4x^2 - 12x + 5\)
\(y(2x - 5) = (2x - 5)(2x - 1)\)

Since \(5/2\) is not an integer, \(2x - 5 \neq 0\) and so we must have \(y = 2x - 1\). So the pairs of positive integers that satisfy the equation are

\[(1, 1), (2, 3), (3, 5), (4, 7), \ldots, (n, 2n - 1), \ldots\]
4. The line through $A$ and $B$ has equation
\[ y - 4 = -\frac{1}{2}(x - 2) \Rightarrow y = -\frac{1}{2}x + 5 \]
which has $x$-intercept $(10, 0)$. Now
\[ A_1 = \frac{1}{2}(10)(4) - \frac{1}{2}(10)(y) = 20 - 5y \]
\[ A_2 = \frac{1}{2}(10)(y) - \frac{1}{2}(10)(2) = 5y - 10 \]

\[ A_1 = A_2 \Rightarrow 20 - y = 5y - 10 \Rightarrow y = 3. \text{ Then} \]
\[ 3 = -\frac{1}{2}x + 5 \Rightarrow x = 4 \quad P(4, 3) \]

5. Area = (Area of triangle $OAC$) + (Area of trapezoid $CABD$) − (Area of triangle $OBD$)
\[ = \frac{1}{2}ab + \frac{1}{2}(b + d)(c - a) - \frac{1}{2}cd \]
\[ = \frac{1}{2}ab + \frac{1}{2}bc + \frac{1}{2}dc - \frac{1}{2}ab - \frac{1}{2}ad - \frac{1}{2}cd \]
\[ = \frac{1}{2}bc - \frac{1}{2}ad = \frac{1}{2}(bc - ad) \]

6. Let the points be $(a, a^2)$ and $(b, b^2)$ with $a < b$. Then the two conditions to be satisfied can be written as
\[ \sqrt{(b - a)^2 + (b^2 - a^2)^2} = 5 \quad \text{and} \quad \frac{b^2 - a^2}{b - a} = \frac{4}{3}. \]
Substituting $b^2 - a^2 = \frac{4}{3}(b - a)$, obtained from the second equation, into the first equation gives
\[ \sqrt{(b - a)^2 + \frac{16}{9}(b - a)^2} = 5 \]
\[ \sqrt{\frac{25}{9}(b - a)^2} = 5 \]
\[ \frac{5}{3}(b - a) = 5 \quad \text{(since} \ a < b \ \text{and hence} \ b - a > 0) \]
\[ b - a = 3 \]

Also, from the second equation we get $b + a = \frac{4}{3}$. Solving these equations simultaneously gives $a = -\frac{5}{6}$ and $b = \frac{13}{6}$. So the two points are $\left(-\frac{5}{6}, \frac{25}{36}\right)$ and $\left(\frac{13}{6}, \frac{169}{36}\right)$. 
7. \( y^2 = x^2 + 2x + 6 \)
   \( y^2 = (x + 1)^2 + 5 \)
   \( y^2 - (x + 1)^2 = 5 \)
   
   \((y - x - 1)(y + x + 1) = 5 \)
   
   \( y - x - 1 = 5 \)  \( y - x - 1 = 1 \)  \( y - x - 1 = -5 \)  \( y - x - 1 = -1 \)
   \( y + x + 1 = 1 \)  \( y + x + 1 = 5 \)  \( y + x + 1 = -1 \)  \( y + x + 1 = -5 \)

The solutions to these four systems of equations are, respectively,

\((-3,3), (1,3), (1,-3), (-3,-3)\)

8. \( y = (x - a)^2 + (x - b)^2 \)
   \( = x^2 - 2ax + a^2 + x^2 - 2bx + b^2 \)
   \( = 2x^2 - 2(a + b)x + a^2 + b^2 \)
   \( \text{ (complete the square) } \)
   \( = 2 \left[ x^2 - (a + b)x + \left( \frac{a + b}{2} \right)^2 \right] + a^2 + b^2 - \frac{(a + b)^2}{2} \)
   \( = 2 \left( x - \frac{a + b}{2} \right)^2 + \frac{a^2 - 2ab + b^2}{2} \)
   \( = 2 \left( x - \frac{a + b}{2} \right)^2 + \frac{(a - b)^2}{2} \)

So the minimum value of \( y \) is \( \frac{(a - b)^2}{2} \).

9. \( y = mx \)
   \( y^2 = (mx - 6)^2 = 4 \)
   \( x^2 + (mx - 6)^2 = 4 \)
   \( x^2 + m^2x^2 - 12mx + 36 = 4 \)
   \( (1 + m^2)x^2 - 12mx + 32 = 0 \)

\( x = \frac{12m \pm \sqrt{144m^2 - 128(1 + m^2)}}{2(1 + m^2)} = \frac{12m \pm \sqrt{16m^2 - 128}}{2(1 + m^2)} \)

For there to be a single solution we must have \( 16m^2 - 128 = 0 \); i.e. \( m^2 = 8 \). And since \( m > 0 \), we must have \( m = 2\sqrt{2} \).

So the equation of the line is \( y = 2\sqrt{2} x \). For the point of intersection we have

\( x = \frac{12(2\sqrt{2})}{2(1 + (2\sqrt{2})^2)} = \frac{24\sqrt{2}}{18} = \frac{4\sqrt{2}}{3} \)

Then \( y = 2\sqrt{2} \left( \frac{4\sqrt{2}}{3} \right) = \frac{16}{3} \). So the point is \( \left( \frac{4\sqrt{2}}{3}, \frac{16}{3} \right) \).

Or Use \( \frac{y-6}{x} = -1 \) along with the equation \( x^2 + (y - 6)^2 = 4 \) to get the same solution.
10. Multiplying through by $ab$ gives

$$b + a^2 + 1 = ab$$

Since $a \neq 1$ (this would give $b + 1 + 1 = b$, which is impossible),

$$a^2 + 1 = b(a - 1)$$

$$b = \frac{a^2 + 1}{a - 1} = a + 1 + \frac{2}{a - 1}$$

Since $a$ and $b$ are positive integers, we must have $a - 1 = 1$ or $a - 1 = 2$; i.e. $a = 2$ or $a = 3$. So the numbers are $a = 2$, $b = 5$ or $a = 3$, $b = 5$. 