THE TWENTY-FIFTH W.J. BLUNDON MATHEMATICS CONTEST*  

Sponsored by  
The Canadian Mathematical Society  
in cooperation with  
The Department of Mathematics and Statistics  
Memorial University of Newfoundland  

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1. Two sides of an isosceles triangle are 5 cm each and the area of the triangle is 12 cm$^2$. Find all possible values for the length of the third side.

2. Solve: $8^x + 16 \cdot 8^{-x} = 17$.

3. If $x^3 + y^3 = 10(x + y)$ and $x^2 + y^2 = 30$, find $xy$.

4. For the points $A(1, 4)$, $B(3, 7)$, $C(7, 8)$ and $D(10, 2)$, find the area of the quadrilateral $ABCD$.

5. A rectangle $ABCD$ has sides $AB = CD = 6$ and $AD = BC = 8$. Two equal circles of radius $r$ are inside this rectangle. One is tangent to $AB$ and to $BC$, and the other is tangent to $CD$ and to $DA$. The two circles are externally tangent to each other. Determine the exact value of $r$.

6. When one kilogram of salt is added to a solution of salt and water, the solution becomes $33 \frac{1}{3}$% salt by mass. When one kilogram of water is added to this new solution, the resulting solution is 30% salt by mass. Find the percentage of salt in the original solution.

7. How many pairs of integers $(x, y)$ satisfy the equation $x^4 + \frac{100}{y^4} = \frac{101x^2}{y^2}$?

8. Show that the circles with equations $x^2 + y^2 + 2x - 8y + 8 = 0$ and $x^2 + y^2 + 10x - 2y + 22 = 0$ are tangent.

9. Find all real numbers $a$ such that the polynomials $x^3 + ax^2 + 1$ and $x^3 + x^2 + a$ have at least one zero in common.

10. Prove that the sum of cubes of three consecutive integers is divisible by 9.

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