Abstracts
of the Talks Presented to the International Workshop
"Locally Finite Lie Algebras"

Banff International Research Station

August 30 - September 5, 2003

1. **Bruce Allison, University of Alberta, Canada**

   Covering Algebras Of Lie Algebras

   In this talk, based on joint work with S. Berman and A. Pianzola, we will discuss loop algebras (also called covering algebras) that are constructed from a base algebra A and a finite order automorphism s of A. (When $s = 1$, the loop algebra is said to be untwisted.) In the case when the base algebra is a Lie algebra, loop algebras have been used to obtain realizations of affine Kac-Moody Lie algebras and, more generally, extended affine Lie algebras.

   In this talk, we describe some methods and results to decide when two loop algebras are isomorphic. These methods include an interpretation of loop algebras as forms of untwisted loop algebras. This interpretation allows us to apply tools from Galois cohomology in the study of the isomorphism problem.

2. **Yuri Bahturin, Memorial University of Newfoundland, Canada and Moscow State University, Russia**

   Introduction To Locally Finite Lie Algebras

   The aim of this talk is to give basic definitions and formulate some general results in the theory of locally finite Lie algebras. We will exhibit several characteristic examples and describe connections with locally finite groups and associative algebras.

3. **Alexander Baranov, University of Leicester, UK**

   Simple Locally Finite Lie Algebras

   Simple locally finite Lie algebras are subdivided into three classes: finitary, diagonal, and non-diagonal. Several characterizations of these classes will be given and various properties of the corresponding algebras will be discussed.

4. **Georgia Benkart, University of Wisconsin - Madison, USA**

   Life On The Wedge (And Superwedge)

   This talk will survey various constructions of Lie algebras and Lie superalgebras using exterior powers. Recent work on Lie (super)algebras graded by finite root systems and locally finite simple Lie (super)algebras will be featured. If time allows, some connections with the Tits construction of exceptional simple Lie superalgebras will be presented.

5. **Roger Bryant, University of Manchester Institute of Science and Technology, UK**

   Free Lie Algebras As Modules For Groups

   I shall describe some recent results concerned with the module structure of a free Lie algebra under the action of a group. Let G be a group, K a field and V a finite-dimensional KG-module. Let $L(V)$ be the free Lie algebra over K which has V as a subspace and every basis of V as a free generating set. The action of each element of G on V extends to a Lie algebra automorphism of $L(V)$. Thus $L(V)$ becomes a KG-module, and each homogeneous component $L^n(V)$ is a KG-submodule, called the nth Lie power of V. We consider the problem of determining the modules $L^n(V)$ up to isomorphism.

6. **Ivan Dimitrov, Queen's University, Canada**

   Irreducible Weight Modules Of $gl(\infty)$

7. **George Elliott, University of Toronto, Canada**

   To Be Announced Later

8. **Helge Glöckner, Technical University of Darmstadt, Germany**

   Direct Limits Of Lie Groups, Topological Groups And Topological Spaces

   An ascending sequence $G_1 \subseteq G_2 \subseteq \cdots$ of finite-dimensional Lie groups is called *strict* if each G_n is closed in G_{n+1}, and equipped with the induced topology. In fundamental investigations by L. Natarajan, E. Rodríguez-Carrington and J.A. Wolf (1991-94), certain conditions on such directed systems had been described which ensure that the direct limit exponential map is sufficiently well-behaved in order to serve as a chart around the identity for an infinite-dimensional Lie group structure on the direct limit group $G = \bigcup_n G_n$. In the first part of the talk, I'll describe an alternative approach which always allows G to be turned into a Lie group, without any extra conditions on the directed system. Instead of using the exponential map, tubular neighbourhoods are used to create compatible families of charts. As an application, in many cases countable-dimensional locally finite Lie algebras can be integrated to Lie groups.

   In the second part of the talk, I'll report on work in progress concerning direct limits of infinite-dimensional Lie groups. I'll consider three typical classes of infinite-dimensional Lie groups which, algebraically, are direct limits of infinite-dimensional Lie groups: 1. countable weak direct products of Lie groups; 2. test function groups; 3. groups of compactly supported diffeomorphisms. Extending earlier work by N. Tatsuuma, H. Shimomura and T. Hirai (1998), I'll discuss the direct limit property of such Lie groups (which may be satisfied or not) in the categories of topological spaces, topological groups, smooth manifolds, and in the category of Lie groups.

9. **Dimitar Grantcharov, University of California-Riverside, USA, and University of Alberta, Canada**

   On The Structure And Characters Of Weight Modules Of Lie Algebras And Superalgebras

   In this talk we will present a method of studying weight modules of Lie superalgebras $\mathfrak{g}$ of type I, and $\mathfrak{g} = \mathbf{W}_n$. The method is based on Mathieu's result that every simple weight $\mathfrak{g}$-module M with finite weight multiplicities is obtained from a highest weight module $L(\lambda)$ by a composition Ψ of a twist and localization. We study the properties of the twisted localization Ψ and relate a Jordan-Hölder series of a highest weight module X with a Jordan-Hölder series of the module $\Psi(X)$. As a main application of the method we reduce the problems of finding a $\mathfrak{g}_0$-composition series and a character formula for all simple weight modules with central character χ to the same problems for simple highest weight modules with the same central character. Some of our results are new already in the case of a classical reductive Lie algebra $\mathfrak{g}$.

10. **David Handelman, University of Ottawa, Canada**

    Classification Of Locally Semisimple Algebras

    This is a survey talk. All algebras are associative. A *locally semisimple* algebra is a union of an increasing family of (finite dimensional) algebras over a field. If the field is algebraically closed, then Elliott's theorem asserts that the naturally ordered Grothendieck group with an additional datum is a complete invariant. The structure of the invariant itself is fairly well understood, completely if the algebra is simple; this is a consequence of Choquet theory and a result of Effros, Handelman & Shen.

When the underlying field is real closed, the classification of locally semisimple algebras is via a triple, corresponding to the two finite dimensional division algebras. If A denotes the algebra and $\mathrm{K}_0(A)$ is ordered Grothendieck group, then a complete invariant is given by the triple $\mathrm{K}_0(A) \to \mathrm{K}_0(A \otimes \mathbf{C}) \to \mathrm{K}_0(A \otimes \mathbf{H})$ as homomorphisms of ordered abelian groups, together with an additional datum. However, the structure for the invariant, that is, the classfication theory for the invariant itself is not completely understood (except when the algebra is simple).

If the underlying field is arbitrary (but perfect), a complete invariant for the algebras is known, but far from well understood. It extends the invariant of the real case, and involves all the finite dimensional division algebras that "appear" in the algebra.

Returning to the case that the underlying field be the complexes, there are analogous classification results for locally semisimple algebras with an action of a group, usually a locally representable action of a compact group. The invariants that result have close connections to random walks and limit ratio results.

11. **Otto Kegel, Freiburg University, Germany**

    Highly Transitive Permutation Groups

12. **Felix Leinen, Johannes Gutenberg University of Mainz, Germany**

    Group Algebras of Simple Locally Finite Groups
    (*joint work with Orazio Puglisi*)

    After an introduction to the various types of simple locally finite groups G we shall discuss the ideal lattice of their group algebra $\mathbb{F}G$ over a field $\mathbb{F}$ of characteristic zero. It was shown by A. E. Zalesskiĭ, that the structure of such an ideal lattice is intimately related to the asymptotic behaviour of the $\mathbb{F}$-representations of the finite subgroups of G.

The ideal lattice turns out to be quite sparse in many cases. On the other hand, some tricky situations remain unsettled, especially when G is *approximated diagonally* by finite alternating groups.

Therefore we shall also discuss the convexly indecomposable normalized positive definite class functions $\mathbb{C}G \longrightarrow \mathbb{C}$ for certain direct limits G of finite alternating groups. These functions can be viewed as analogues of complex characters for G.

13. **Karl-Hermann Neeb, Technical University of Darmstadt, Germany**

    Approximating Infinite-Dimensional Lie Groups By Locally Finite Ones

    In the representation theory of infinite-dimensional Lie groups the following phenomenon occurs in many interesting situations: One is interested in an infinite-dimensional Lie group G containing a directed union of finite-dimensional subgroups which either is dense or at least "determines" in a certain sense the representations one is interested in. This establishes an interested link between certain direct limits of finite-dimensional groups and certain groups of operators on Hilbert spaces. The approximation by the finite-dimensional groups is crucial to determine the topology of the large group, its central extensions and, to some extent, also its representations

14. **Erhard Neher, University of Ottawa, Canada**

    Locally Finite Root Systems

    Locally finite root systems are defined in analogy to the definition of a finite root system, except that the finiteness condition is replaced by local finiteness, i.e., the intersection of the root system with every finite-dimensional subspace is finite. A theory of locally finite root systems has recently been developed by Ottmar Loos and the speaker (Weyl groups, bases, classification, parabolic subsets, positive systems, weights). In the talk, the basic structure theory will be presented. We will also consider Lie superalgebras graded by locally finite root systems, and give a characterization of the locally finite ones.

15. **Ivan Penkov, University of California - Riverside, USA**

Recent Advances In Locally Finite Lie Algebras And Superalgebras And Some Unsolved Problems

I will describe a class of semisimple locally finite Lie superalgebras admitting a root decomposition (classically semisimple locally finite Lie superalgebras) which contains the class of root reductive locally finite Lie algebras, and in particular $gl(\infty)$. Then I will describe all Cartan subalgebras of $gl(\infty)$. Finally, I will describe the current state of the theory of weight modules over root-reductive locally finite Lie algebras. Throughout the talk, I will state open problems.

16. **Arturo Pianzola, University of Alberta, Canada**

Loop Algebras.

The (twisted) loop algebras of finite dimensional simple complex Lie algebras were first considered by V. Kac to provide concrete realizations of the affine Kac-Moody algebras. I will begin by describing a procedure that allows one to view loop algebras in general in terms of (algebraic) principal homogeneous spaces. Several examples will be given to illustrate this point (which is very natural and geometric in nature).

The bulk of the talk will be centered around joint work with B. Allison and S. Berman on iterated loop algebras and related applications to the study of Extended Affine Lie Algebras.

If time permits, I will briefly talk about some applications of loop algebras to the classification of conformal algebras.

17. **Pavel Shumyatski, University of Brasilia, Brazil**

Positive Laws In Fixed Points

18. **Helmut Strade, Hamburg University, Germany**

Locally Finite Lie Algebras Over Fields Of Arbitrary Characteristic.

19. **Joe Wolf, University of California, Berkeley, USA**

Double Fibration Transform

The double fibration transforms considered here, carry cohomology of holomorphic vector bundles to spaces of holomorphic functions, in a manner equivariant for the action of a semisimple Lie group. The best-known example is the complex Penrose transform. In the last year there has been a lot of progress on the general theory for the double fibration transform from holomorphic vector bundles on a flag domain. This talk is an indication of the current state of the theory.

20. **Alexander Zalesskii, University of East Anglia, UK**

Classification Of Simple Infinite Dimensional Lie Subalgebras Of Locally Finite Associative Algebras

This is a joint work with A. A. Baranov and Yu. A. Bahturin. The main result is a kind of classification of simple Lie subalgebras in locally finite associative algebras over complex number field. In fact, we reduce the problem to the classification of simple associative algebras with involution as follows.

Let L be a simple Lie subalgebras (under the bracket multiplication) of locally finite associative algebra A. Then either L is of finite dimension or there exists an associative algebra B with involution $*$ such that L is isomorphic to the commutator subalgebra $[U, U]$ of the Lie algebra U of $*$-skew symmetric elements of B (that is, $U = \{b \in B : b^* = -b\}$. Additionally, B has no non-zero proper $*$-stable ideal. In general B is not an envelope of L in A.